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TORSIONAL STRENGTH AND BEHAVIOR OF CONCRETE BEAMS IN COMBINED LOADING

by

GANPAT SHANKAR PANDIT, B.E.(Honours), M.Sc.

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

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FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "TORSIONAL STRENGTH AND BEHAVIOR OF CONCRETE BEAMS IN COMBINED LOADING" submitted by GANPAT SHANKAR PANDIT in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

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ABSTRACT

The object of the investigation was to study the behavior of plain and reinforced concrete beams subjected to various combinations of bending, torsion and shear. The study was especially directed to controversial issues and some of the variables not examined previously.

The experimental work included in this investigation comprised of design and fabrication of a setup for applying combined loading and tests on 36 plain and reinforced concrete beams of 6 x 12-in. nominal size. Twenty beams were tested in different combinations of bending and torsion. Various combinations of reinforcing steel were used including the case of a typical beam section in which most of the longitudinal steel is located in the zone of flexural tension as well as the case in which the longitudinal steel is equally distributed near the top face and the bottom face of the beam. Sixteen beams were tested in different combinations of bending, torsion and shear. All the beams tested in this investigation were first subjected to transverse load and then twisted to failure.

Based primarily on the observations made in the experimental part of this investigation, a simple analysis has been developed for predicting the torsional strength of a beam subjected to combined loading. The analysis is direct and general, its validity extending to the limiting case of pure torsion.

Using the analysis presented in this investigation, the analytical torsional strengths of 166 beams are compared with the actual torsional

strengths. Sixty-eight of these beams were tested by the author and 98 by other investigators.

An expression for the initial torsional stiffness of a beam subjected to combined bending and torsion has been developed and compared with the test results.

The tests indicate that the presence of flexure to a certain limit increases the torsional strength of a typical beam section. For these sections, the flexural stresses have the effect of a favourable prestress as far as torsional strength is concerned. This is not so for the case of a beam with equal longitudinal steel on the two faces. For these beams, the presence of flexure generally reduces the torsional strength. The torsional strength of beams is not reduced due to the presence of transverse shear provided enough transverse steel exists to prevent a shear failure in the absence of torsion. The initial torsional stiffness is reduced due to the presence of flexural moment.

The analysis reproduces the actual trends in the variation of torsional strength as affected by flexure. It also provides a means of obtaining the upper bound and the lower bound of the actual torsional strength.

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LIST OF SYMBOLS AND DEFINITIONS

All the symbols are explained when they are first introduced. They are collected here for ready reference.

Cross-sectional Constants and Dimensions

b_o = overall width of rectangular section

d_o = overall depth of rectangular section

d = effective depth; depth from the top face to the center of bottom longitudinal steel

jd = internal lever arm

d_1 = depth of compressed concrete

d_2 = depth of non-compressed concrete

b' = width of the tie on center lines of the legs

d' = depth of the tie on center lines of the legs

s = spacing of the ties

a_1 = concrete cover over the center line of the bottom longitudinal steel

a_2 = concrete cover over the center line of the top longitudinal steel

a_3 = concrete cover over corner longitudinal bars on the sides

R = radius of the Mohr circle

r = distance of a longitudinal bar from the axis of the beam

A_s = area of the bottom longitudinal steel

A_s' = area of the top longitudinal steel

a_v = area of cross-section of one leg of the tie

Stresses

f'_c = compressive strength of concrete determined from 6 x 12-in. control cylinders

Cu = compressive strength of concrete determined from 6-in. control cubes

f'_t = tensile strength of concrete in direct tension

f'_t (indirect test) = tensile strength of concrete determined from indirect tensile test on 6 x 12-in. cylinders

τ_o = unit torsional strength of plain concrete in pure torsion

τ = apparent unit torsional strength of plain concrete under combined stresses

σ = normal compressive stress

τ_{oct} = octahedral stress

σ_1, σ_2, σ_3 = three principal stresses

σ_c = nominal compressive strength of concrete

I_1, I_2, I_3 = three stress invariants

τ_1 = apparent unit torsional strength of compressed concrete

τ_2 = apparent unit torsional strength of non-compressed concrete

τ_m = maximum possible apparent unit torsional strength of plain concrete under combined stresses

σ_o = optimum value of the normal stress

σ_{cr} = critical normal stress at which the failure changes from cleavage (tension) to shear (compression) type under combined stresses

f_y = yield stress of the bottom longitudinal steel

f'_y = yield stress of the top longitudinal steel

f_{yt} = yield stress of the transverse steel

f_s = stress in the bottom longitudinal steel due to flexure

f'_s = stress in the top longitudinal steel due to flexure

f_{st}	=	stress in the transverse steel
v_c'	=	unit transverse shearing strength of concrete
v_u	=	nominal ultimate shear stress due to transverse shear in combined loading
	=	$\frac{V_u}{b_o d}$

<u>Forces and Moments</u>

C	=	total flexural compression
F	=	lateral force developed by a longitudinal bar due to torsion
V_u	=	ultimate shear force in combined loading
S.F	=	shear force
Tu	=	ultimate torque of a reinforced concrete section in combined loading
Tuo	=	ultimate torque of a reinforced concrete section in pure torsion
T_c	=	torsional strength of concrete in combined loading
T_{co}	=	torsional strength of plain concrete section in pure torsion
T_{c1}	=	torsional strength of compressed concrete
T_{c2}	=	torsional strength of non-compressed concrete
T_s	=	total torsional resistance of steel
T_{s1}	=	contribution of the transverse steel to torsional strength
T_{s2}	=	contribution of the longitudinal steel to torsional strength
Mu	=	ultimate bending moment in combined loading
Muo	=	assumed ultimate moment in pure bending
	=	$0.9d\ A_s f_y$ for reinforced concrete section
	=	$\frac{1}{4}\ b_o d_o^2 f_t'$ for plain concrete section

m_p = fully plastic moment of a longitudinal bar

Tuo^a = ultimate torque of a reinforced concrete section in pure torsion computed from the analysis.

Tuo^t = ultimate torque of a reinforced concrete section in pure torsion determined by test.

Tu^a = ultimate torque of a reinforced concrete section in combined loading computed from the analysis

Tu^t = ultimate torque of a reinforced concrete section in combined loading determined by test

B.M = bending Moment

T.M = twisting Moment

Dimensionless Factors

n = modular ratio

j = ratio of length of internal lever arm to the effective depth

$k_1 = \frac{M_u}{M_{uo}}$

k_p = coefficient for the fully plastic section in torsion

k_e = coefficient for the elastic section in torsion

n_1 = number of ties intersected by a potential failure crack on the longer side of the rectangle

n_2 = number of ties intersected by a potential failure crack on the shorter side of the rectangle

θ_1 , θ_2 = slopes of the sand heap in modified sand heap analogy

k and m = constants in the Equations $\tau_o = f_t' = k(f_c')^m$

k_2 = τ_2/τ_o

θ_o = angle of twist per unit length per unit torque for a reinforced concrete beam subjected to pure torsion

θ = angle of twist per unit length per unit torque for the reinforced concrete beam subjected to combined bending and torsion

$p \quad = A_s/b_o d_o$

$p' \quad = A'_s/b_o d_o$

$p_t \quad = \dfrac{\text{volume of transverse steel}}{\text{gross volume of concrete}}$

Miscellaneous

S = initial torsional stiffness of a beam subjected to combined bending and torsion

S_o = initial torsional stiffness of the beam subjected to pure torsion

Definitions

Axis of a beam of rectangular cross-section is defined as the straight line parallel to the length of the beam passing through the intersection of the diagonals of the rectangle.

Compressed concrete is defined as the portion of concrete which is compressed due to the flexural moment.

Non-compressed concrete is defined as the portion of concrete which is not compressed due to flexure.

Failure is defined as the stage at which the torsional load dropped suddenly and beyond which an increase in the angle of twist did not correspond to an increase in the twisting moment.

Apparent unit torsional strength is defined as the unit torsional strength in combined stresses. The word apparent distinguishes the unit torsional strength for the particular stress condition from the intrinsic strength of the material.

CHAPTER I

INTRODUCTION

1-1 Introductory Remarks

Very few tests have been carried out on concrete in torsion combined with flexure and transverse shear as compared to those in pure torsion. Even pure torsion in concrete has not been studied as extensively as flexure and transverse shear. There appear to be two main reasons for this:

(i) It is argued that torsional stresses, in general, are only of secondary nature compared to those due to flexure or transverse shear and

(ii) torsion tests require special equipment.

However, the above reasons can hardly justify the lack of research on torsion especially when the limits of application of concrete are being pushed back continuously.

The previous research on torsion, especially in combination with bending and shear, has left much to be desired. There is disagreement on some of the vital aspects of the problem of reinforced concrete subjected to combined loading. This is evident from the following brief discussion.

The pioneers in the field of torsion in concrete directed their tests essentially towards the verification of Saint Venant's classical theory of torsion. Later investigations (Nylander, 1945 and Gardner, 1960),

however, have indicated that the classical elastic theory is unsuitable for estimating the ultimate torsional strength of concrete. Both Nylander and Gardner believed that better results could be obtained by assuming full plasticity (Nadai, 1931, 1950). Cowan (1950), Ernst (1957) and Zia (1961), however, felt that full plasticity could not be attained due to the limited extent of plastic redistribution of stresses. It might, therefore, seem that the elastic theory underestimates and the plastic theory overestimates the torsional strength. The error associated with either theory depends probably upon the extent of redistribution of stresses due to plasticity. No method for determining the extent of redistribution of stresses is yet available.

Nylander (1945) argued that in common practice the torsional stresses in concrete are only of secondary nature. He considered his beams, which had no transverse steel, as typical load resisting elements. It might, however, appear questionable whether Nylander's beams were really typical in view of the common practice of providing nominal transverse steel even in ordinary flexural members. Nylander's tests and conclusions were limited to the less common case of beams without transverse steel.

Cowan (1953) has developed equations representing the interaction of bending and torsion in plain, reinforced and prestressed concrete beams. His approach is based on the elastic theory. His equations can not be used beyond the visco-elastic limit which marks the end of the elastic action.

Gesund and Boston (1964) have presented expressions for torsional strength under combined loading. Their method involves a trial and error solution which is cumbersome. Besides, the theory developed does not

explain the fact that the torsional strength can be increased due to the presence of bending moment (Nylander, 1945 and Cowan and Armstrong, 1955).

Cowan (1953) and Cowan and Armstrong (1955) have presented interaction diagrams for combined bending and torsion. The interaction diagram, based on their test results for reinforced concrete beams, is in conflict with the findings of Chinenkov (1959) and Gesund, Schuette, Buchanan and Gray (1964) in respect of flexural strength. The latter investigators have reported that the flexural strength can only be reduced due to the presence of torsion whereas tests by Cowan and Armstrong showed increase in flexural strength due to torsion. The theoretical interaction diagrams presented by Cowan (1953) have validity restricted to the visco-elastic limit.

From the above discussion it appears that the problem of combined loading is still far from solution. It is believed that adequate understanding of the behavior of beams under combined loading can result only from extensive investigations, both theoretical and experimental. The present tests, together with the related analyses, are an attempt to understand more fully the behavior of plain and reinforced concrete beams under combined loading.

1-2 Object

The main objectives of the investigation are as follows:

(1) To study the behavior of plain and reinforced concrete beams subjected to combined loading.

(2) To study the effect of flexure and transverse shear on the torsional strength of beams with various amounts and arrangements of reinforcing steel.

(3) To study the interaction of flexure, torsion and transverse shear for beams with various combinations of reinforcing steel.

(4) To develop an analysis to predict the torsional strength of beams subjected to combined loading and to compare the analytical results with the test results in this investigation as well as with the test results of other investigations.

1-3 Scope

The experimental work included in this investigation consisted of tests on 36 plain and reinforced concrete beams subjected to various combinations of bending, torsion and transverse shear.

The nominal cross-section of all beams was 6 x 12-in. There were 9 plain concrete beams and 27 beams with longitudinal bars and ties.

Twenty beams were tested in various combinations of bending and torsion. Of these beams, five were of plain concrete and the rest were reinforced so as to represent typical beam sections and sections designed essentially for torsion. The beams were divided into five groups. The reinforcing steel for beams in the same group was identical.

Sixteen beams were tested in various combinations of bending, torsion and shear. Of these beams, four were of plain concrete and the rest had both longitudinal bars and ties. The beams were divided into four groups. The reinforcing steel for beams in the same group was identical.

All beams were first subjected to the transverse load and then twisted to failure. The beams within each group were tested using varying levels of the transverse load. In general, the first beam in a group was tested with a high transverse load producing stresses in the bottom steel

close to yield stress. The transverse load was successively reduced for other beams within the group and the last beam was tested with no transverse load (pure torsion).

Based primarily on the observations made in the experimental part of this investigation, a simple analysis has been developed for predicting the torsional strength of a beam subjected to combined loading. The analysis is direct and general, its validity extending to the limiting cases.

Using the analysis developed in this investigation, the analytical torsional strengths of 166 beams have been compared with the actual torsional strengths. Sixty-eight of these beams were tested by the author and 98 by other investigators.

An expression for the initial torsional stiffness of a beam subjected to combined bending and torsion has been developed and compared with the test results.

CHAPTER II

REVIEW OF PREVIOUS WORK

2-1 Introduction

There have been relatively few tests on concrete (plain, reinforced or prestressed) subjected to torsion and bending with or without transverse shear. Tests on concrete in combined stresses have been reported by Nylander (1945), Cowan and Armstrong (1955), Chinenkov (1959), Lialin (1959), Gardner (1960), Gesund and Boston (1964) and Gesund, Schuette, Buchanan and Gray (1964). Besides Cowan (1953) and Lessig (1959) have presented analytical studies of the problem of combined bending and torsion.

The above tests and investigations have left much to be desired. There is disagreement on some vital points. Besides some of the many variables involved have never been investigated. In the following sections, the work of the previous investigators has been briefly reviewed.*

2-2 Tests on Plain Concrete

Nylander (1945) tested 16 plain concrete beams in axial compression and torsion. The compressive force was applied first and then the specimens were twisted to failure. As expected, the cracking and ultimate torques were equal. However, the torsional strength was increased

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*This review deals only with combined loading. Pure torsion has been adequately reviewed by Kemp, Sozen and Siess (1961) and Pandit (1963).

considerably due to the axial compression. The applied compressive stresses had essentially the same effect as uni-axial prestress which had to be overcome before torsional failure could occur. All 16 beams tested by Nylander had compressive stress less than 50% of the cube strength of concrete and presumably failed due to diagonal tension. The inclination of the failure cracks to the axis decreased with increasing values of compressive stress. Nylander preferred the plastic theory to predict the ultimate strength in combined compression and torsion.

Cowan and Armstrong (1955) tested 3 plain concrete beams, the first in pure torsion, the second in pure bending and the third in combined bending and torsion with a bending moment to torque ratio of 2. The failure, which was sudden and destructive, occurred at the formation of first crack. Cowan (1953) reported that in pure torsion or in combined bending and torsion, the failure occurred with a cleavage fracture. Based on this, he developed an expression for the visco-elastic limit of plain concrete beams.

2-3 Tests on Reinforced Concrete

(i) Beams with Longitudinal Reinforcement only

Nylander (1945) considered torsion only as a secondary effect. Hence all his reinforced beams had only longitudinal steel. Forty-four beams of square, rectangular and 'T' section were tested in various combinations of bending, torsion and shear. Tests on 34 beams in combined bending and torsion showed that the torsional strength was increased by applying bending moment not exceeding the flexural cracking moment. On the basis of his tests on 10 beams in combined torsion and transverse

shear, Nylander stated that the ultimate capacity of the member may be determined by equating the sum of torsional and shearing stresses to the ultimate tensile strength of concrete.

Nylander's investigation comprised of six main divisions. Part I dealt with beams subjected to pure torsion. He considered the classical theory set up by Saint Venant unsuitable and preferred the plastic theory to predict ultimate torsional strength. Part II dealt with the influence of normal compressive stresses on the torsional capacity. Parts III and IV examined the effects of flexure and shear respectively on the torsional capacity of beams. Cases of torsional restraint in structures were studied in Parts V and VI.

Gesund and Boston (1964) reported tests on 10 beams with square and rectangular cross-section. Two beams were tested in pure torsion and the rest in combined bending and torsion. The purpose of the study was to determine the failure mechanism and to find a way of predicting the ultimate resistance under combined loading. The total resistance to torsion was assumed to consist of dowel action of longitudinal reinforcement and the resistance of uncracked portion of concrete. It was not found possible to write an expression for the latter part of the total resistance to torsion. Using a theoretical model and a series of assumptions, an expression was derived for the torsional resistance due to the dowel action. The calculated torque capacities obtained by using the derived expression were lower than the experimental ultimate torques for all beams reported in the investigation. It was concluded that in the absence of transverse reinforcement, the dowel action of the longitudinal steel was predominant in resisting torsion. No expression for flexural capacity under combined

loading was derived. However, it was stated that the flexural capacity might be reduced by the torsion due to the great reduction in the lever arm. It was also stated that in combined loading, the beams may fail in one of the several different torsional modes or in bending.

(ii) <u>Beams with Longitudinal and Transverse Reinforcement</u>

Cowan and Armstrong (1955) tested 7 beams with rectangular cross-section in combined bending and torsion. It was believed that the primary bending failure was associated with vertical flexural cracks and crushing of the compressed concrete whereas primary torsion failure was associated with the development of 45^{o} helical cracks. The tests indicated that the torsional capacity of the beams was increased due to the bending moment which induced compression in concrete on the top. This compression had to be overcome before failure could occur.

Cowan (1953) presented analyses which could be used upto visco-elastic limit but not for precise assessment of ultimate capacity under combined loading. Cowan believed that the torsional resistance moment of a beam in the elastic range was given by the sum of the resistances of the concrete and the reinforcement.

It was observed that the introduction of small amount of bending increased the torsional moment while the addition of small amount of torsion reduced bending resistance only slightly. Cowan, therefore, felt that it might be reasonable to design a reinforced concrete section independently for bending and torsion without reduction in the maximum permissible stresses. This was equivalent to the assumption that the actual interaction diagram for combined loading might be simplified to one consisting of a pair of straight lines parallel respectively to the

bending and torque axes.

Cowan developed equations expressing the interaction of bending and torsion using maximum stress, maximum strain and internal friction theories. The validity of these equations was restricted to the visco-elastic limit. He also gave a graphical representation of the relation between bending and torsion at the visco-elastic limit. This limit was assumed to represent the end of the elastic action and beyond which large scale cracking occurred.

Chinenkov (1959) reported tests on 36 beams in combined bending and torsion. The purpose of the first series, comprising 23 beams, was to examine the feasibility of reinforcing concrete elements subjected to combined bending and torsion by only two plane welded skeletons placed near the sides. The second series, comprising 13 beams, was devoted to a verification of the design formulas proposed by Lessig (1959). In these tests attention was mainly devoted to cases in which the ratio of torsion to bending moment was low.

It was reported that diagonal cracks developed in the zone of flexural compression before failure occurred. The component of the principal tensile stresses parallel to the axis of the beam was resisted by the longitudinal reinforcement while the component in the lateral direction was resisted by the transverse reinforcement. In the absence of transverse reinforcement on two of the faces in the first series, the component in the lateral direction tended to bend the longitudinal steel and cause premature failure.

The cracking torque was reported to be about the same as for plain concrete in pure torsion. Measurements of strains in the vertical

legs of the stirrups showed that they developed appreciable stress only when cracks intersected the concrete zone in which the stirrups were located. Whenever a stirrup intersected a failure crack, the stress in it reached the yield point. The torsional stiffness was higher for lower flexural moment, higher percentage of longitudinal and transverse steel and higher concrete strength.

The failure in all cases was preceded by diagonal cracks due to torsion on the face compressed in flexure. Such cracks could not appear under combined loading at a value of twisting moment less than that corresponding to first cracking in pure torsion. In the case of beams of second series, which had closed stirrups, the failure was preceded not only by diagonal cracks on top but also by the yielding of the reinforcement.

A precise determination of the depth of compressed zone or the position of neutral axis was not found feasible due to the splitting of concrete in flexural compression zone. However, it was felt that the flexural capacity is reduced due to the coexisting torsion.

The test results were compared with values computed using Russian code Ni Tu 123-55 and formulas proposed by Lessig (1959). There was reasonable agreement with the Russian Code for low values of torsional moment but there was appreciable divergence for higher torsional moments. A fair correlation with Lessig's formulas was reported. The somewhat higher test loads were attributed to the contribution of uncracked concrete in the portion containing flexural cracks.

Gesund, Schuette, Buchanan and Gray (1964) reported tests on 12 reinforced concrete beams of square and rectangular section in combined bending and torsion. The variables were the concrete strength, the amount

and spacing of transverse reinforcement and the moment torque ratios. The test results were compared with values obtained from a theoretical model developed from observed failure mechanisms. A fair correlation was claimed.

It was believed by the investigators that the flexural capacity can only be reduced by the coexisting torque. The reduction in flexural capacity was attributed to (i) the bending couple produced by the force in the ties cutting the failure surface, (ii) the dowel action of longitudinal reinforcement and (iii) the reduction of the lever arm. While expressions to deal with (i) above were developed, effects listed as (ii) and (iii) above were ignored for the cases in which bending predominated failure. To deal with the cases in which torsion predominated failure, the formula proposed by Gesund and Boston (1964) was modified to include the dowel action of stirrups in the torsional resistance. It was assumed that the stirrups could work either in tension or as dowels. Two independent expressions for the torsional resistance were developed. The first was based on the assumption that the entire torsional resistance was produced by the tension in the ties. In the second expression, the total resistance to torsion was taken as the sum of the dowel effects in the longitudinal reinforcement and the stirrups. The ultimate torque was taken as the greater of the two values obtained from the two expressions.

The investigators concluded that even relatively small amounts of transverse reinforcement could lead to bending rather than torsional failure for torque moment ratios less than half. They also believed that, provided sufficient transverse reinforcement was used to prevent torsional failure, the stresses in the transverse reinforcement had no effect on the flexural strength of the beam.

2-4 Tests on Prestressed Concrete

Tests on prestressed concrete in combined bending and torsion have been reported by Cowan and Armstrong (1955) and Gardner (1960).

Cowan and Armstrong reported tests on 3 uniformly prestressed and 6 eccentrically prestressed beams in various combinations of bending and torsion. The beams had no transverse reinforcement. The failure occurred under combined loading as soon as the first crack formed. The failure was sudden and destructive. There was almost no plastic detrusion. In this regard, the behavior was similar to that of plain concrete beams. However, large increases in strength compared to plain concrete beams were reported.

The procedure used by Cowan (1953) for determining the twisting moment at which the first crack occurs in combined loading was similar to that for plain concrete. He derived a formula expressing the interaction of bending and torsion. This formula could not be used for beams which showed extensive cracking since the validity of the expression was restricted to the visco-elastic limit.

Gardner (1960) reported tests on 16 prestressed concrete 'I' beams subjected to combined bending and torsion. Bending moment, varying from 10 to 80 percent of flexural capacity of the beam, was first applied and the specimen then twisted to failure.

In contrast to the beams tested by Cowan and Armstrong, the beams tested by Gardner showed considerable ductility after cracking. This difference in behavior was attributed to the shape of the cross-section. The stiffness of the beam was believed to drop when the web cracked due to high shearing stresses in it. However, the failure could not be

precipitated until the flanges were also stressed to the ultimate tensile strength of concrete.

Gardner found that the magnitude and the direction of the principal tensile stresses and consequently the orientation of the cracks depended on the level of prestress. The plastic theory was preferred for predicting the ultimate capacity whereas the elastic theory could be used to get a fair estimate of the behavior in the elastic range. The tests indicated that the torsional capacity was not appreciably affected unless the level of bending was very high.

CHAPTER III

SPECIMENS AND TESTING PROCEDURE

3-1 Test Specimens

Thirty-six beams, reported in this investigation, were divided into Groups A through H. There were 9 beams in Group A, 3 in Group E and 4 in each of the other groups. All beams had a nominal cross-section of 6 x 12-in. Beams in the same group had identical reinforcement. The beams belonging to the same group were distinguished by the numeral following the group designation (for example B-1, B-2, B-3 and B-4). The same mix design for concrete was used for all the beams.

The first five beams in Group A and all the beams in Groups B, C, D and E were designed for tests in combined bending and torsion. The last four beams in Group A and all the beams in Groups F, G and H were designed for tests in combined bending, torsion and shear. The beams in Groups A through E had an overall length of 10 feet 2 inches whereas beams belonging to Groups F, G and H had an overall length of 5 feet 4 inches.

The details of the reinforcing steel are shown in FIGURE 3-1 and TABLE 3-1. Group A consisted of plain concrete beams. Beams in all the remaining groups had both longitudinal bars and ties. The longitudinal steel for the beams belonging to Groups B, C and D was designed so as to cover the range from under-reinforced to over-reinforced beams from the point of view of conventional working stress design. The amount of longitudinal steel for beams in Group E was designed to be about the same as

for beams in Group C. However, for beams belonging to Group E, the longitudinal steel was equally divided on the top and the bottom faces. The transverse steel was identical for the beams in Groups C, D and E.

All beams in Groups F, G and H had about the same amounts of longitudinal and transverse steel. However, the size of tie bar and consequently the spacing was different in Groups F and G. The beams in Groups G and H had identical transverse steel. The amount of total longitudinal steel was also the same for these two groups. However, in Group H all the longitudinal steel was placed on the bottom face whereas in Group G it was equally distributed on the top and the bottom faces as shown in FIGURE 3-1 and TABLE 3-1.

All the specimens had additional steel outside the gage length. This ensured that the beams failed inside the gage length. The gage lengths for the beams are shown in FIGURES 3-2 and 3-3.

The properties of sand, coarse aggregate and reinforcing steel are given in Section A-1 and the fabrication of specimens is described in Section A-2 of Appendix A. The compressive and tensile strengths of concrete were determined from compression and indirect tensile tests on 6 x 12-in. control cylinders. These are given in TABLES 4-1 and 4-2, CHAPTER IV.

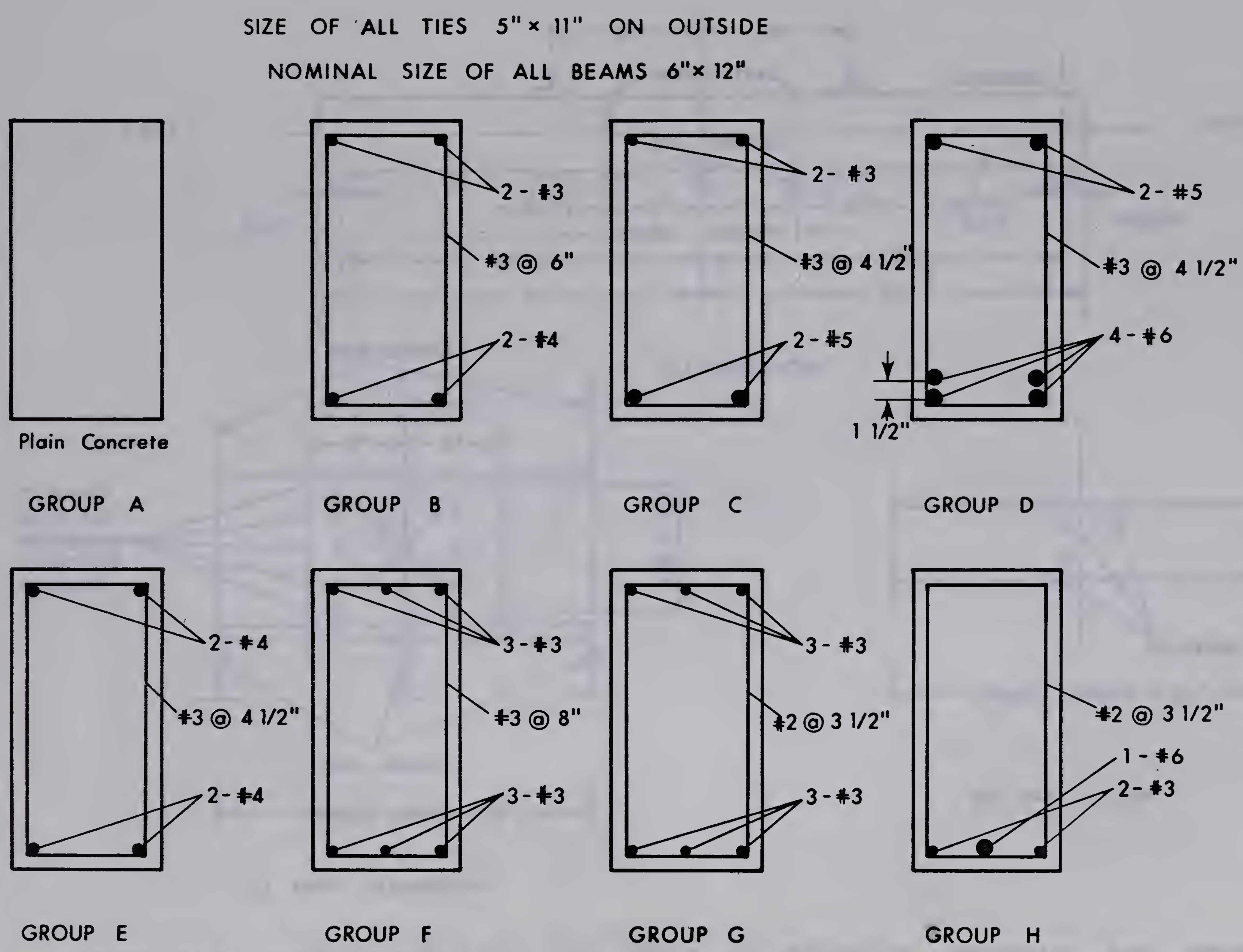
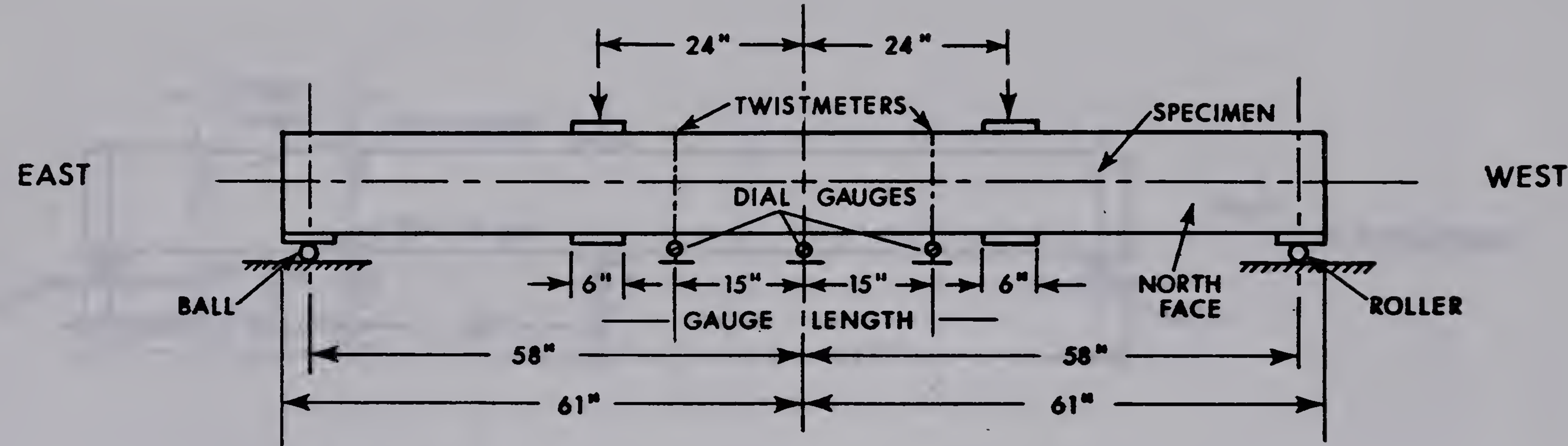
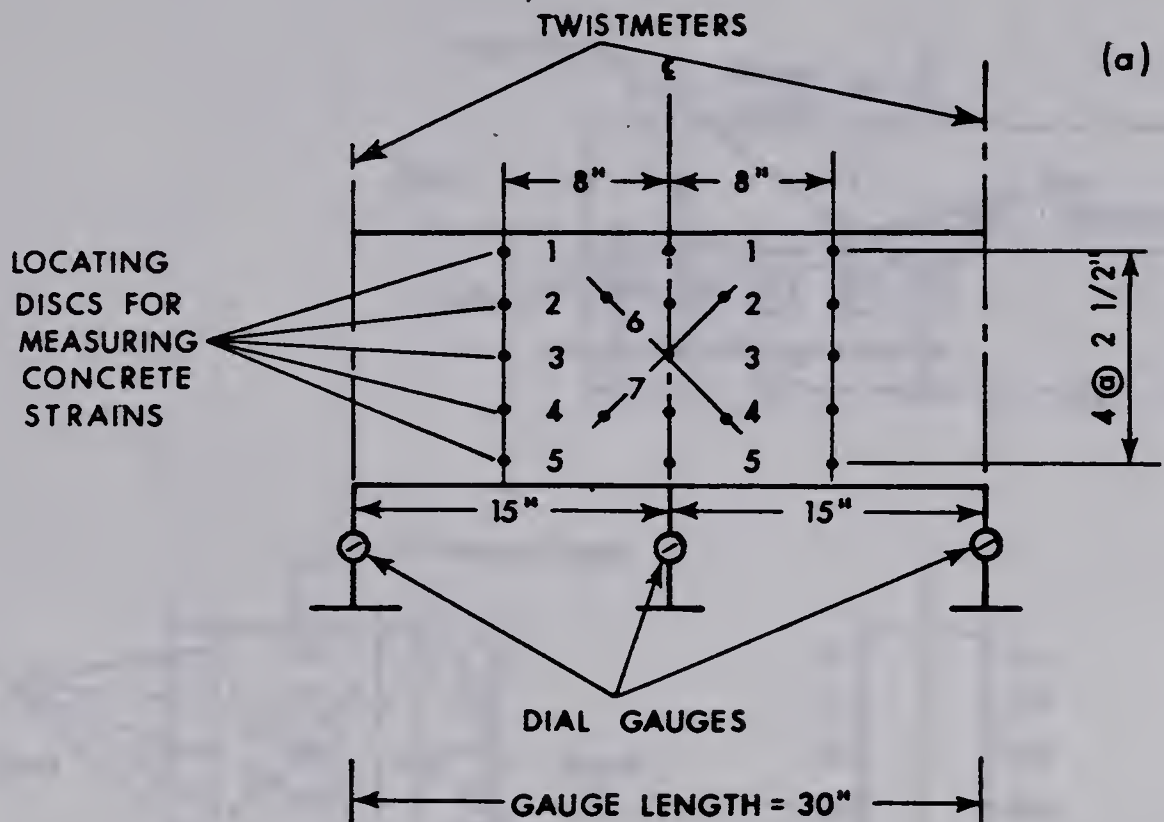


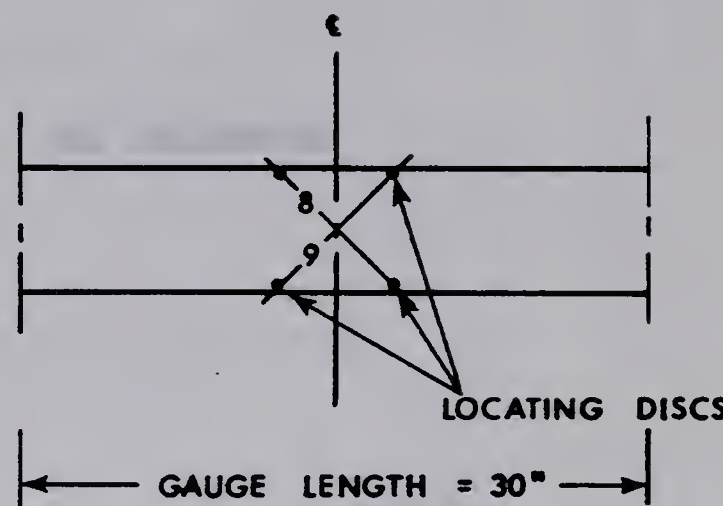
FIGURE 3-1 DETAILS OF REINFORCING STEEL



(a) ELEVATION



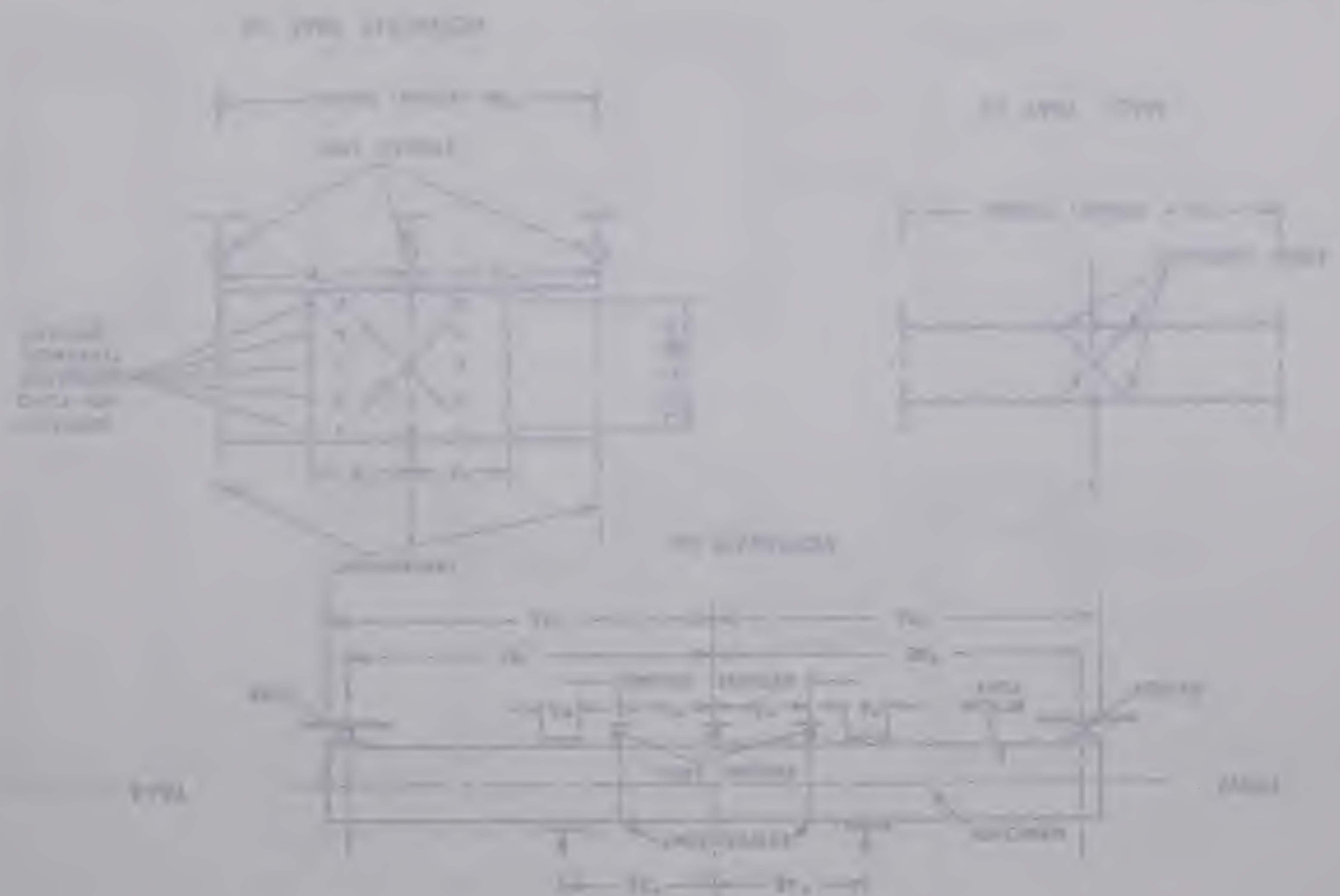
(b) PART ELEVATION



(c) PART PLAN

FIGURE 3-2 INSTRUMENTATION FOR TESTS IN COMBINED BENDING AND TORSION

FIGURE 3-1 [illegible]



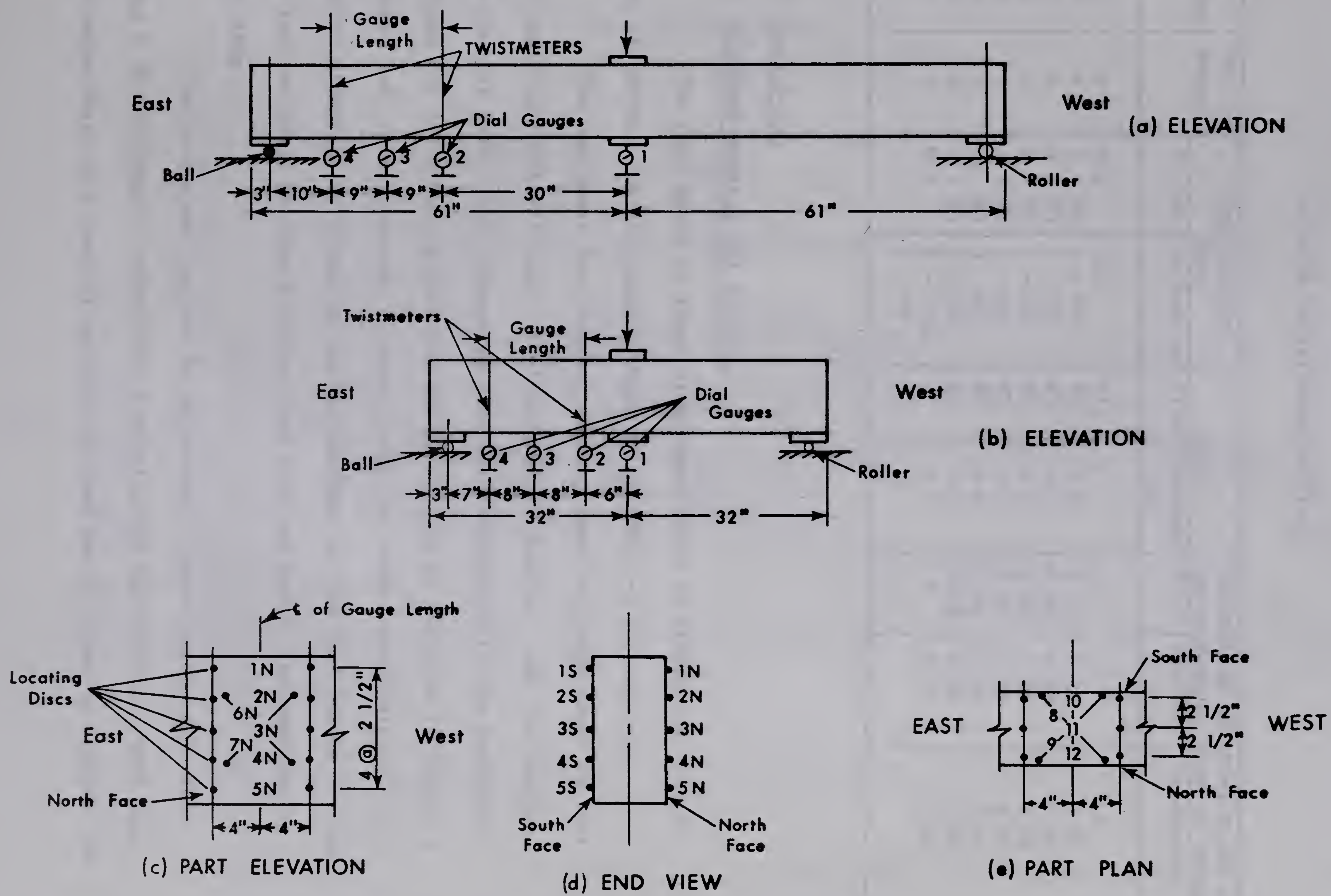


FIGURE 3-3 INSTRUMENTATION FOR TESTS IN COMBINED BENDING, TORSION AND SHEAR

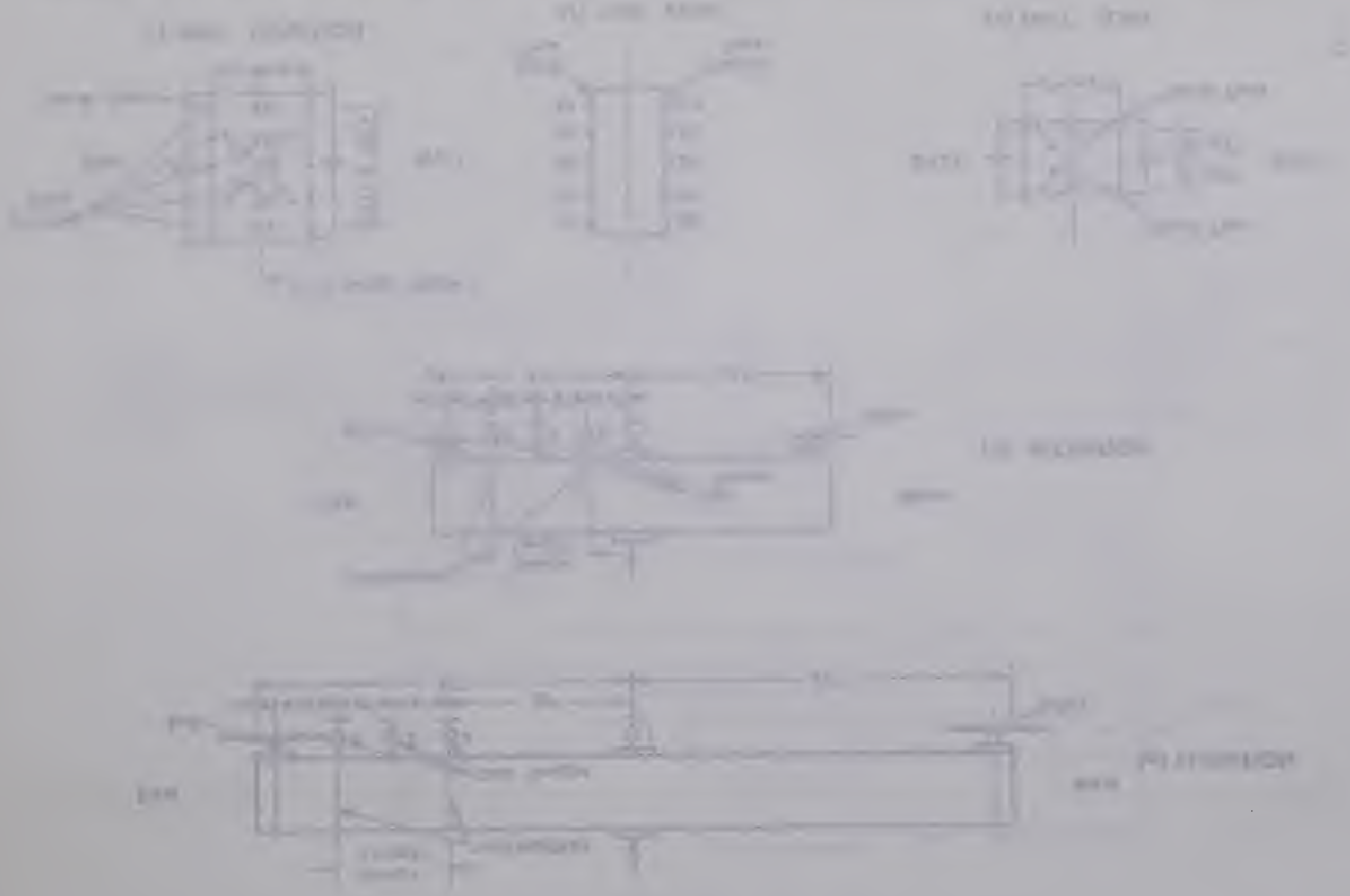


TABLE 3-1

DETAILS OF REINFORCING STEEL

Group	No. of Beams	Longitudinar Bars		Ties		Percentage of steel based on gross nominal area of concrete		
		At top	At bottom	Size	Spacing in.	Top	Bottom	Transverse
A	9	Nil	Nil	Nil	-	0	0	0
B	4	2 - #3	2 - #4	#3	6.0	0.31	0.55	0.82
C	4	2 - #3	2 - #5	#3	4.5	0.31	0.83	1.09
D	4	2 - #5	4 - #6	#3	4.5	0.83	2.45	1.09
E	3	2 - #4	2 - #4	#3	4.5	0.55	0.55	1.09
F	4	3 - #3	3 - #3	#3	8.0	0.46	0.46	0.61
G	4	3 - #3	3 - #3	#2	3.5	0.46	0.46	0.62
H	4	Nil	2 - #3 & 1 - #6	#2	3.5	0	0.92	0.62

3-2 Test Setup

(i) Combined Bending and Torsion

The test setup is shown diagrammatically in FIGURE 3-4. It is also illustrated by photographs shown in FIGURES 3-5 through 3-9.

The equipment for applying bending load was independent of that for applying torsional load. A 44-kip Amsler jack was used for applying the bending load. The transverse load was transferred to the specimen through the distributing beam and set of rollers as shown in FIGURES 3-4 through 3-6. Six-inch wide circular rings cut from a steel pipe (16-in. outside diameter) were mounted on the specimen at the load points so that the torsion could be freely transmitted from one end of the specimen to the other.

The torque was applied by stressing a three-fourth-inch cable using a 30-ton Simplex center-hole hydraulic ram operated by a hand pump. The center-hole jack reacted against the load bed as shown in FIGURES 3-4

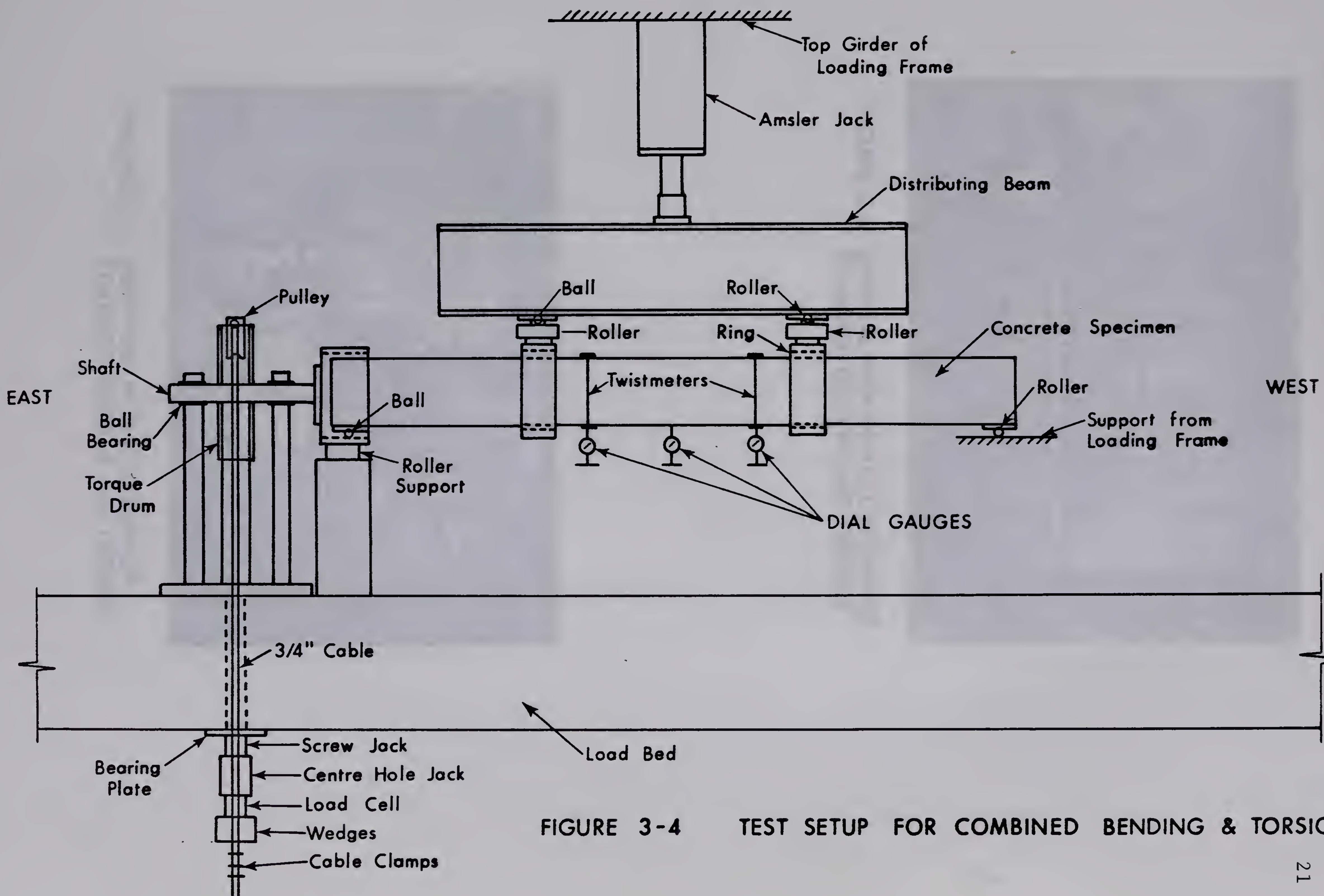
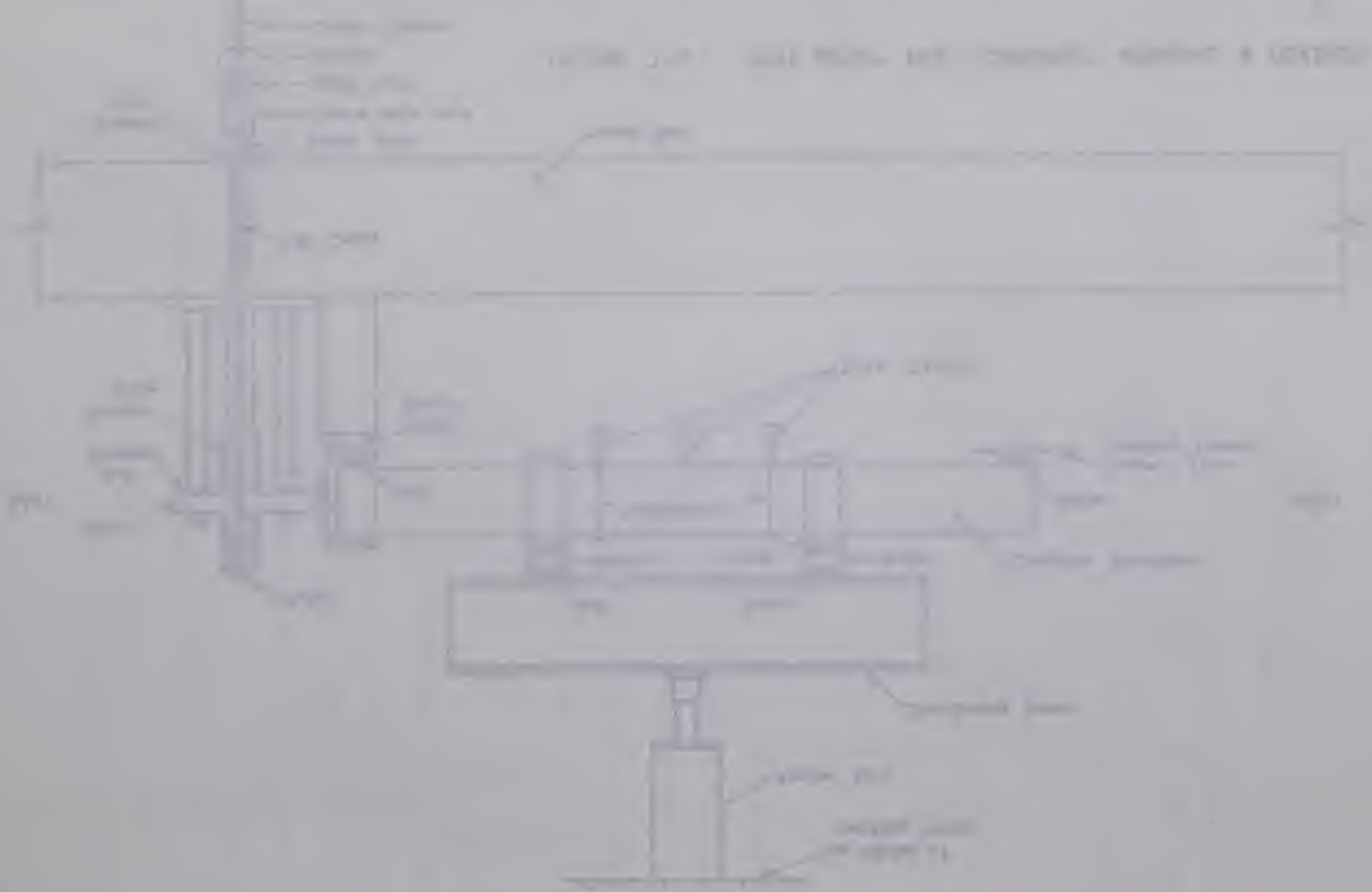


FIGURE 3-4 TEST SETUP FOR COMBINED BENDING & TORSION



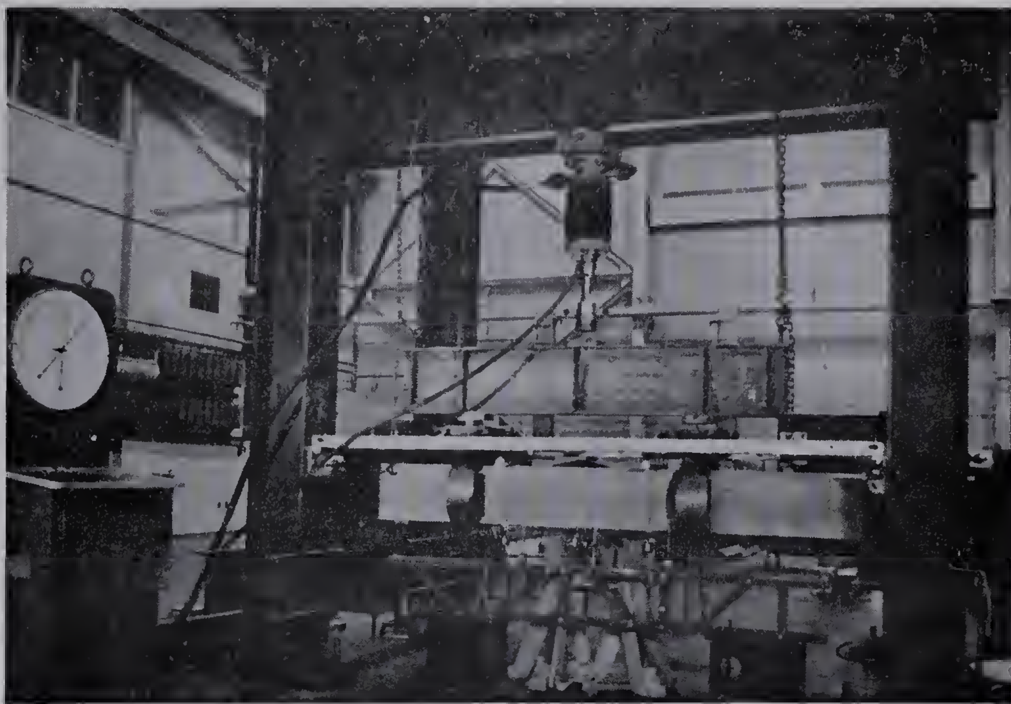


FIGURE 3–5 GENERAL VIEW OF TEST SETUP FOR COMBINED BENDING AND TORSION

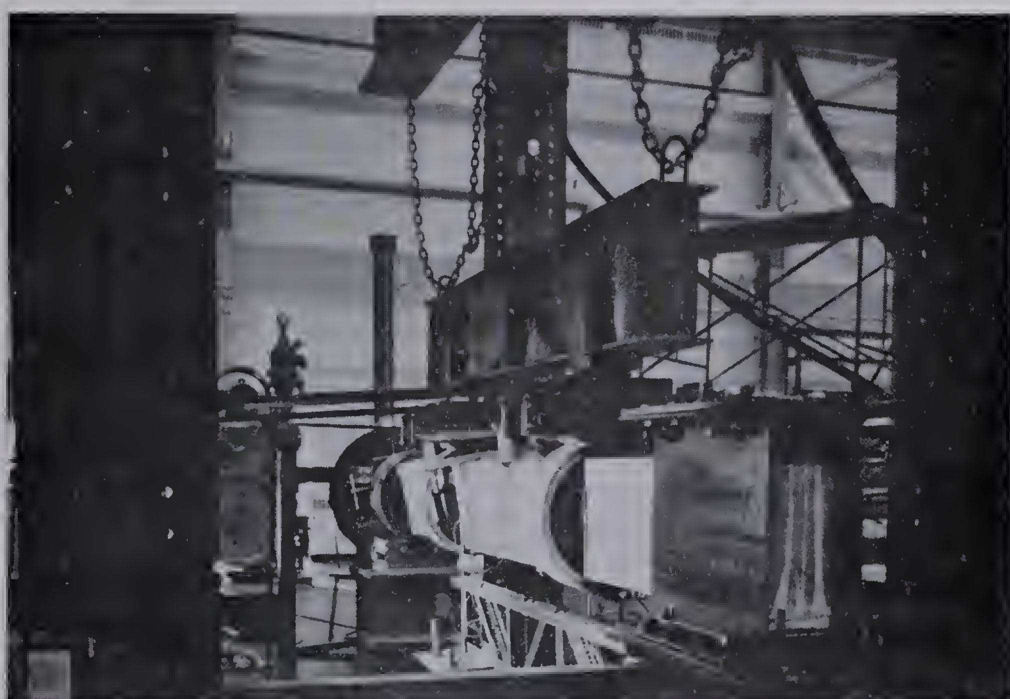


FIGURE 3–6 SIDE VIEW OF TEST SETUP FOR COMBINED BENDING AND TORSION

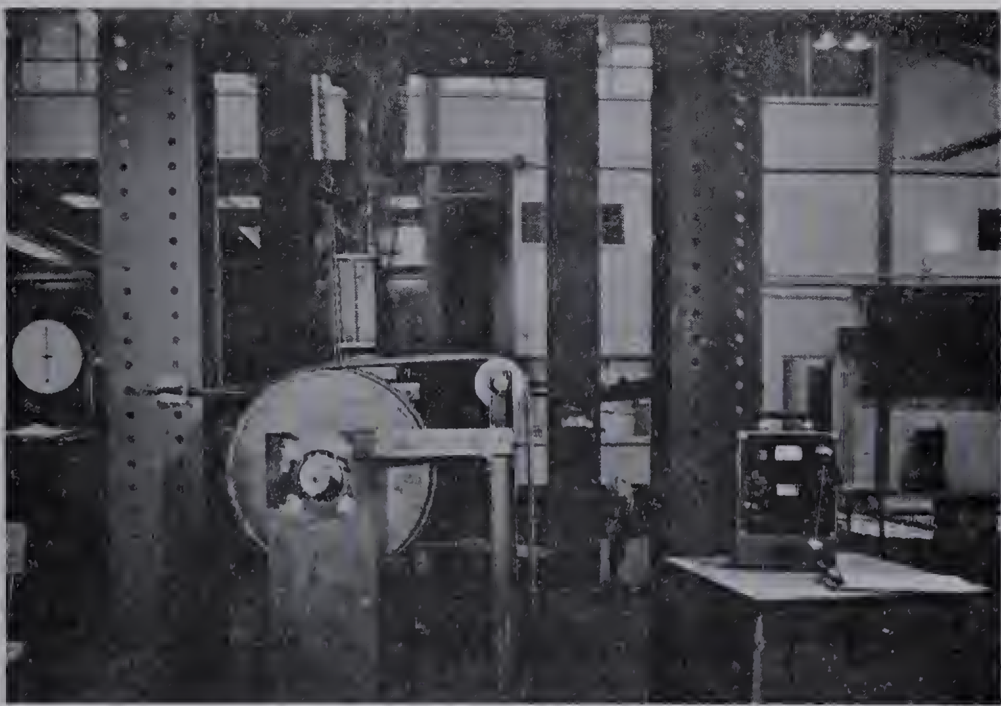


FIGURE 3–7 END VIEW OF TEST SETUP FOR COMBINED BENDING AND TORSION

FIGURE 3–8 DETAILS OF TWISTING END

FIGURE 3–9 CABLE TENSIONING DEVICE

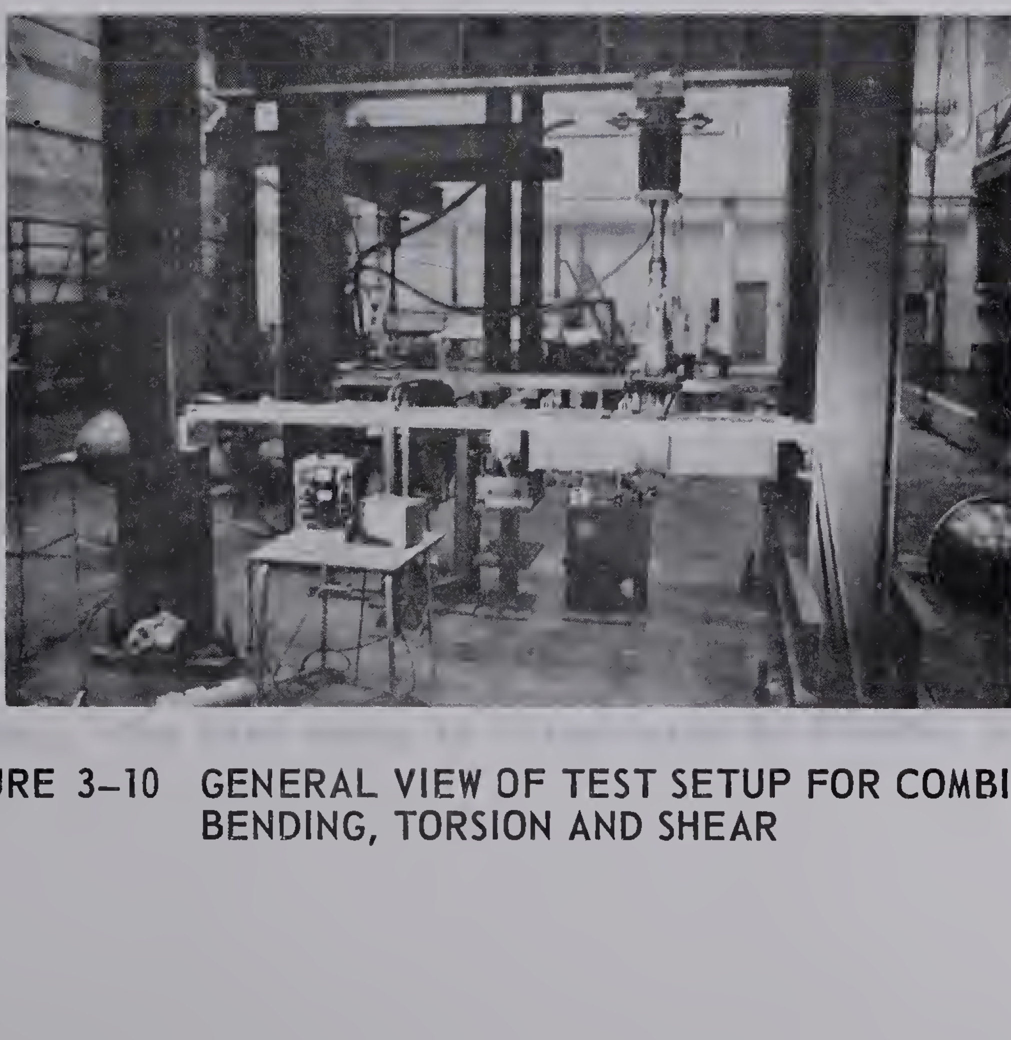


FIGURE 3–10 GENERAL VIEW OF TEST SETUP FOR COMBINED BENDING, TORSION AND SHEAR

and 3-9. The cable was placed round a 24-in. torque drum as shown in FIGURES 3-7 and 3-8. The torque drum transmitted the torque to the specimen through a 3-in. diameter shaft. The tension in the cable was determined by means of a load cell and an indicator. The load cell and the indicator were calibrated using a 200-kip universal testing machine. To provide a check on the torque computed from the indicator reading, the torque transmitted by the shaft was determined by means of SR-4 electrical strain gages mounted on the shaft in the direction of the principal stresses. The shaft and the pulley used for changing the direction of the cable had ball-bearing supports, as shown in FIGURE 3-7, to minimize friction. However, the torque lost due to friction was calculated using a previously calibrated device. This device consisted of a three-fourth-inch steel rod used for providing the resisting torque. The stress-strain characteristics of the rod were determined using SR-4 electrical resistance gages and Demec mechanical strain gage. The difference of the torque applied by the cable and the resisting torque provided by the rod was taken as the torque lost due to friction. This loss was allowed for in computing the torque transmitted to the specimen.

(ii) <u>Combined Bending, Torsion and Shear</u>

The test setup for combined bending, torsion and shear was similar to that for tests in combined bending and torsion except that the transverse load was applied at midspan as shown in FIGURES 3-3 and 3-10. The transverse load was applied by means of a 44-kip Amsler jack through a roller and a ring. The roller and the ring were mounted at midspan of the specimen. The test setup is illustrated by FIGURES 3-3 and 3-10.

3-3 Testing Procedure

The transverse load was applied first. The level of the transverse load was the highest for the first beam in a group and decreased for subsequent beams. The last beam in each group was tested in pure torsion. After the desired transverse load was applied, it was held constant and the specimen was twisted to destruction. The constant load maintaining feature of the Amsler machine was utilized to ensure a constant transverse load throughout the twisting operation. The specimen was simply supported throughout the stages involving increments of the transverse load only. When the desired transverse load was attained, the ends were clamped and the twisting was commenced. The twisting operation was continued well beyond the peak value of the twisting moment. After each load increment was applied, the load was held constant for taking the readings and marking the crack pattern.

3-4 Instrumentation of the Test Specimens

In addition to the measurement of the transverse load and the twisting moment, the specimens were instrumented for the measurement of the following:

(i) Concrete Strains: The strains in concrete were measured on both vertical faces and on the top of the specimen. The positions of the locating discs for measuring the concrete strains are shown in FIGURES 3-2 and 3-3. All longitudinal and diagonal strains were measured on a 8-in. gage length.

(ii) Steel Strains: SR-4 electrical strain gages were used for the measurement of steel strains. The strains in the longitudinal bars and the ties were measured.

(iii) <u>Angle of Twist:</u> The twistmeters* were mounted at each end of the gage length as shown in FIGURES 3-2 and 3-3. The difference of the readings of the two twistmeters indicated the net angle of twist over the gage length.

(iv) <u>Deflections:</u> The locations of the dial gages for the measurement of deflections are shown in FIGURES 3-2 and 3-3.

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* For detailed description, see Pandit (1963).

CHAPTER IV

PRESENTATION OF TEST RESULTS

4-1 Principal Test Results

The observations at the end of every load increment for the specimens tested are reported in detail in Appendix B. Separate plots for individual beams are also presented in Appendix B. This chapter deals with the principal test results and the general behavior of the specimens. The main test results are reported in TABLES 4-1 and 4-2. TABLE 4-1 deals with the tests in combined bending and torsion and TABLE 4-2 deals with the tests in combined bending, torsion and shear.

4-2 Behavior of the Test Specimens

As stated in Section 3-3, the transverse load was applied first and the specimens were then twisted to failure. The failure was defined as the stage at which the torsional load dropped suddenly. Beyond this stage an increase in the angle of twist did not correspond to an increase in the twisting moment. The ultimate torque or the torsional strength was defined as the peak torque resisted by the specimen. The specimens were twisted well beyond the ultimate torque.

(i) Combined Bending and Torsion

Group A

Beams A-1 through A-5 in Group A were tested in various combinations of bending and torsion. As expected of plain concrete beams, the

TABLE 4-1

PRINCIPAL TEST RESULTS: COMBINED BENDING AND TORSION

Beam	Age at Test	Concrete Strength		Actual Size		Maximum Bending Moment	Maximum Torque	Maximum Twist	Steel Strains					
									At Commencement of twisting			Near Peak Torque*****		
		Compr-essive*	Ten-sile**	Width	Depth				Bottom Bar	Tie	Top Bar	Bottom Bar	Tie	Top Bar
	days	psi		in.		in. kips		Radians per inch x 10^6	Micro-inches per inch					
A-1	31	4340	433	6.25	12.20	59	0	--	--	--	--	--	--	--
A-2	43	4790	460	6.50	12.20	42	53	33.3	--	--	--	--	--	--
A-3	50	4500	469	6.10	12.20	35	53	66.6	--	--	--	--	--	--
A-4	48	5200	460	6.05	12.20	25	52	57.8	--	--	--	--	--	--
A-5	50	5200	460	6.10	12.20	0	58	91.1	--	--	--	--	--	--
B-1	42	4560	467	6.20	12.20	**** 220	--	--	--	--	--	--	--	--
B-2	46	4640	406	6.20	12.20	195	72	910	2000	150	--	4470	1705	--
B-3	49	4690	397	6.05	12.20	110	95	1255	640	-145	--	4230	2960	--
B-4	48	5070	462	6.10	12.20	0	85	1305	0	0	0	2020	1050	4450
C-1	50	4980	442	6.00	12.20	280	78	443	1985	170	*** -505	2545	660	*** -300
C-2	54	4820	427	6.15	12.20	195	105	1035	1300	185	-520	4780	1805	725
C-3	49	5340	502	6.10	12.20	110	111	1545	670	45	-270	1925	515	1075
C-4	54	5800	497	6.50	12.20	0	111	1190	0	0	0	1200	1465	2790
D-1	57	4880	449	6.20	12.20	637	99	1530	9350	140	-5685	10620	1860	-11435
D-2	56	5100	441	6.20	12.20	365	164	1755	--	65	-765	--	1565	295
D-3	59	4650	441	6.20	12.20	195	156	1860	525	90	-475	1210	2025	1580
D-4	62	5080	416	6.20	12.20	0	146	2170	0	0	0	795	1360	2345
E-1	56	4910	381	6.20	12.20	195	76	532	2140	165	-600	10140	1910	-130
E-2	58	5100	438	6.20	12.20	110	101	1425	1020	105	-365	1910	1215	1200
E-3	59	4700	422	6.20	12.20	0	121	2080	0	0	0	3460	1090	870

* Based on compressive test on 6 x 12-in. cylinders
** Based on indirect tensile test on 6 x 12-in. cylinders
*** Minus sign denotes compression
**** Test discontinued
***** The last recorded strains

TABLE 4-2

PRINCIPAL TEST RESULTS: COMBINED BENDING, TORSION AND SHEAR

Beam	Age at Test	Concrete Strength		Actual Size		**** Maximum Bending Moment	Maximum Torque	Maximum Shear Force	Maximum Twist	Steel Strains					
										At Commencement of twisting			Near Peak Torque*****		
		Compr-essive*	Ten-sile**	Width	Depth					Bottom Bar	Tie	Top Bar	Bottom Bar	Tie	Top Bar
	days	psi		in.		in. kips		kips	Radians per inch x 10^6	Micro-inches per inch					
A-6	98	5200	460	6.20	12.20	33	52	1.6	51.8	--	--	--	--	--	--
A-7	94	4850	418	6.10	12.20	41	40	2.0	148	--	--	--	--	--	--
A-8	99	3740	367	6.20	12.20	24	43	1.1	51.8	--	--	--	--	--	--
A-9	100	3750	306	6.20	12.20	0	42	0	59.2	--	--	--	--	--	--
F-1	74	5060	384	6.10	12.20	140	58	9.2	876	1640	830	*** -145	2110	2430	210
F-2	78	5060	384	6.10	12.20	95	83	6.2	815	780	240	-120	2015	2610	1740
F-3	84	4650	370	6.15	12.20	50	89	3.2	1545	165	80	-125	2770	3470	2190
F-4	85	4650	370	6.10	12.20	0	95	0	3330	0	0	0	2730	3950	2155
G-1	88	4560	357	6.20	12.20	155	73	10.2	1520	2115	400	-180	2580	2540	1240
G-2	89	4560	357	6.20	12.20	95	92	6.2	1480	1190	270	-140	2510	5220	1770
G-3	85	4920	348	6.15	12.20	50	103	3.2	1505	105	0	- 80	2465	7220	2290
G-4	84	4920	348	6.20	12.20	0	115	0	2680	0	0	0	2340	8720	2710
H-1	89	4450	352	6.10	12.20	245	74	16.2	1020	9920	2590	--	13300	9160	--
H-2	91	4450	352	6.10	12.20	155	84	10.2	975	1240	380	--	2030	5620	--
H-3	92	4830	350	6.25	12.20	80	88	5.2	982	410	160	--	1570	1420	--
H-4	93	4830	350	6.25	12.20	0	64	0	108	0	0	--	330	1320	--

* Based on compressive test on 6 x 12-in. cylinders
** Based on indirect tensile test on 6 x 12-in. cylinders
*** Minus sign denotes compression
**** Average over the gage length
***** The last recorded strains

failure in all cases was sudden. The deflections and the angles of twist at failure were very small.

Group B

The beams belonging to this group exhibited appreciable yielding. Beam B-1 was tested in pure bending, Beams B-2 and B-3 in combined bending and torsion and Beam B-4 in pure torsion. The higher level of transverse load on Beam B-2 as compared to that for Beam B-3 delayed the appearance of diagonal torsion cracks on the top face which was under flexural compression. However, the failure did not occur until the diagonal cracks due to torsion had appeared on the top face. The inclination of the diagonal cracks on the top face with the axis of the beam was somewhat less for Beam B-2 than that for Beam B-3.

Group C

The four beams in this group were tested with decreasing ratios of flexure to torsion. All beams showed appreciable yielding. The cracking was extensive before failure. The diagonal cracks appeared on the top face in all cases before failure. The inclination of these cracks with the axis of the beam increased as the level of the bending load was reduced. For Beam C-4, tested in pure torsion, the helical cracks were inclined at about 45° to the axis of the beam.

Group D

The beams in this group showed great toughness and yielding. Due to the fairly high amount of tension steel for the beams in this group, it was possible to apply greater flexural load on these beams as compared to the flexural load applied to beams in other groups. The maximum flexural load was applied to Beam D-1. The application of torsion to this beam

did not produce diagonal cracks on the top face. Instead it precipitated crushing and complete destruction of the zone in flexural compression. However, Beams D-2 and D-3, which were subjected to smaller flexural loads, failed after the development of diagonal cracks on the top face due to torsion. Beam D-4, which was tested in pure torsion, had extensive helical cracking prior to failure.

Group E

The three beams in this group were tested with successively decreasing flexural loads. All three beams developed diagonal cracks on top face before failure. A higher flexural load tended to delay the appearance of these cracks and to reduce their inclination to the axis of the beam. All beams exhibited extensive cracking, yielding and toughness before failure.

(ii) <u>Combined Bending, Torsion and Shear</u>

Group A

Beams A-6 through A-9 were tested in various combinations of bending, torsion and shear. These plain concrete beams failed suddenly. Deflections and the angles of twist at failure were very small.

Group F

All beams in this group showed considerable yielding. The flexural load for all of the beams in this group was not large enough to precipitate crushing at failure. Hence, failure was preceded by formation of diagonal cracks on top face. Beam F-1 had fairly wide flexure-shear cracks and the stress in the tension steel was near the yield point before twisting was commenced. On the South face the shearing stresses due to transverse shear added to those due to torsion whereas on the North face

they tended to cancel each other. Hence the diagonal cracks on the South face preceded those on the North face. Beams F-2 and F-3, which were subjected to smaller flexural load, failed after formation of diagonal cracks on the top face. Beam F-4, which was tested in pure torsion, had extensive helical cracks prior to failure.

Group G

The behavior of the beams in this group was similar to that of the corresponding beams in Group F. All beams developed diagonal cracks on the top face before failure. The inclination of these cracks to the axis of the beam decreased for higher transverse load. All beams were extensively cracked prior to failure. The beams in this group developed higher ultimate torques than those developed by corresponding beams in Group F due to the closer spacing of ties for the beams in this group.

Group H

Beam H-1 had a fairly large transverse load (32 kips) and wide flexure-shear cracks before twisting was commenced. The corresponding large flexural compression on the top face inhibited the appearance of diagonal cracks due to torsion. The failure was precipitated as soon as the diagonal cracks appeared on the top face. Beams H-2 and H-3, which were subjected to smaller transverse loads, showed diagonal cracks on the top face well before the ultimate torque was attained. Beams H-1, H-2 and H-3 showed considerable yielding. Beam H-4, which was tested in pure torsion, exhibited very limited yielding. There were only a few cracks until the ultimate torque was reached. The failure occurred rather suddenly.

4-3 Crack Patterns

(i) Combined Bending and Torsion

The crack patterns for the beams at the end of tests in combined bending and torsion are illustrated in FIGURES 4-1 through 4-4. One beam from each group has been chosen for illustration. The crack patterns were similar on the two vertical faces of each beam. In the zone of flexural compression, the angle of inclination of the diagonal cracks due to torsion with the axis of the beam was smaller for beams with higher transverse load as compared to that of beams with lower transverse loads. The diagonal cracks due to torsion either branched off from the flexural cracks or appeared to cross them. In each case, torsion produced new cracks inclined at about 45^{o} to the axis of the beam in the portion cracked previously by flexural tension. The flexural tension cracks were fairly close to vertical in the constant moment region. The numbers appearing on the illustrations refer to the stages of loading as shown in the corresponding tables in Appendix B.

(ii) Combined Bending, Torsion and Shear

The crack patterns for tests in combined bending, torsion and shear are illustrated in FIGURES 4-5 through 4-8. The crack patterns on the two vertical faces of the same beam were not similar due to the effect of transverse shear. The shearing stresses due to torsion and transverse shear were of the same sense on the South face. Cracking on this face was more extensive as compared to that on the North face. Besides all cracks followed the same general direction on the South face. On the North face, however, the shearing stresses due to transverse shear produced diagonal tension in a direction normal to that due to torsional

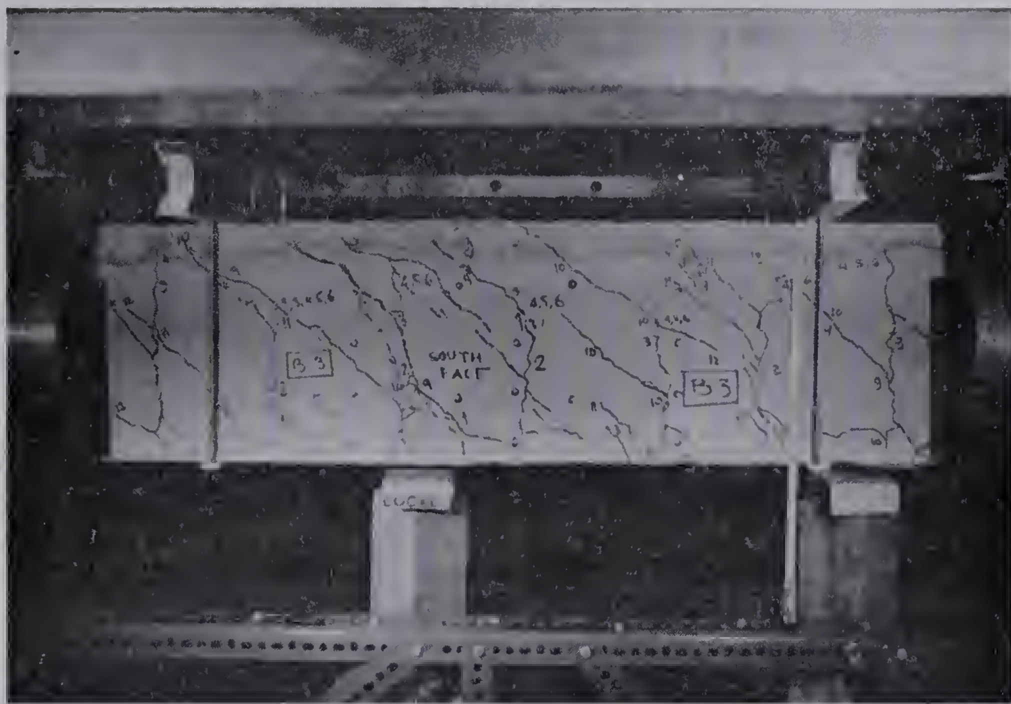


FIGURE 4–1 CRACK PATTERN FOR SOUTH FACE OF BEAM B–3

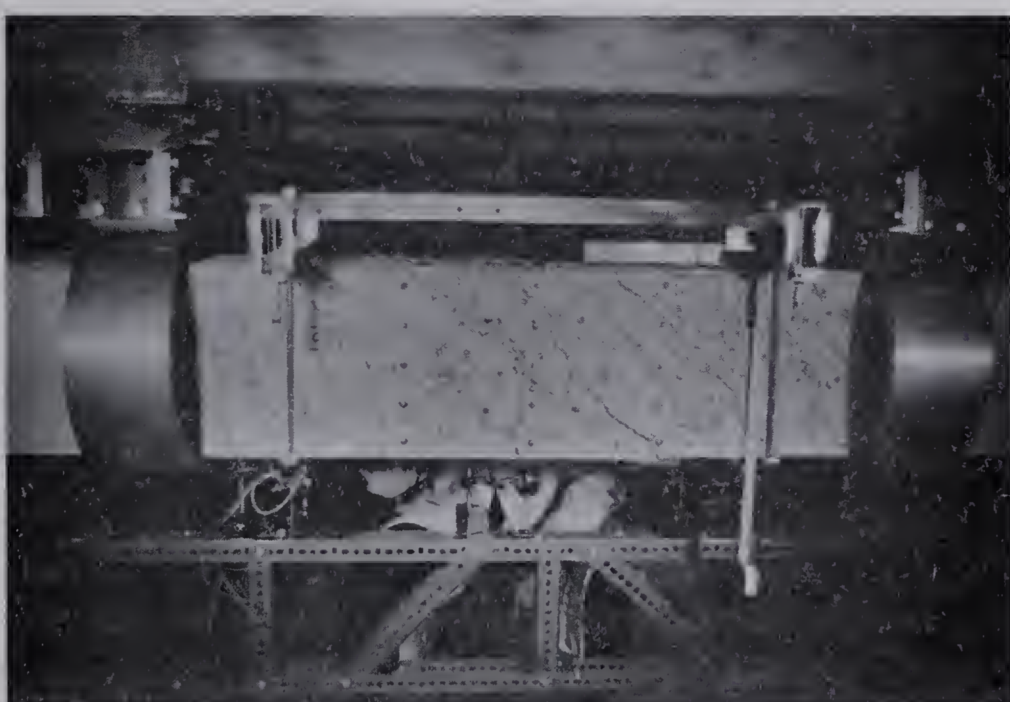


FIGURE 4–2 CRACK PATTERN FOR NORTH FACE OF BEAM C–2

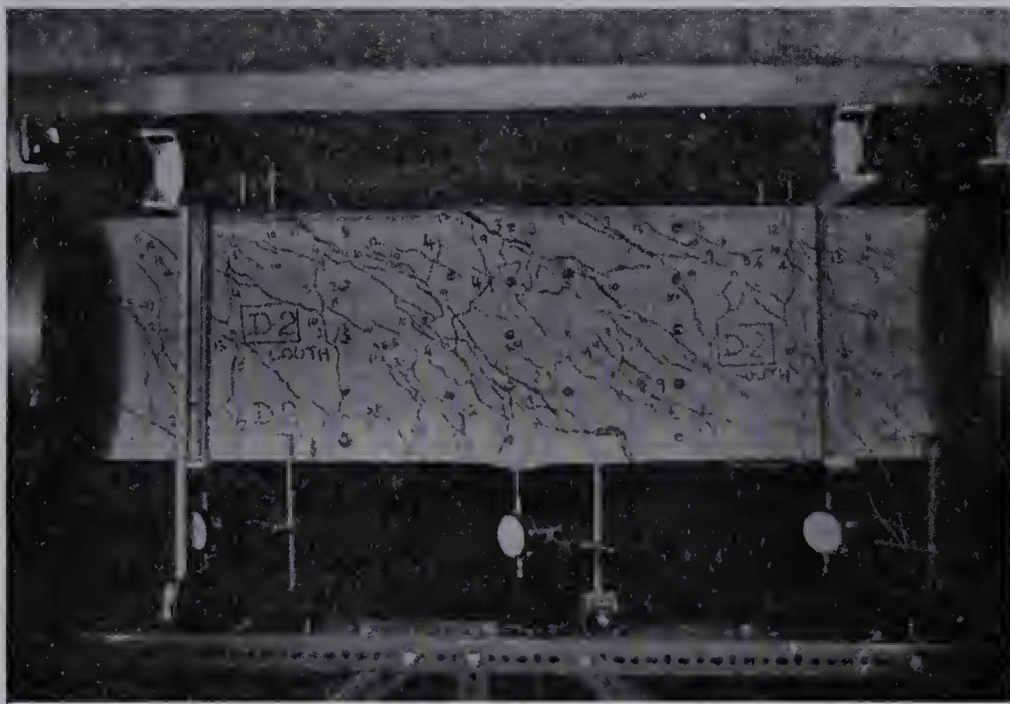


FIGURE 4–3 CRACK PATTERN FOR SOUTH FACE OF BEAM D–2

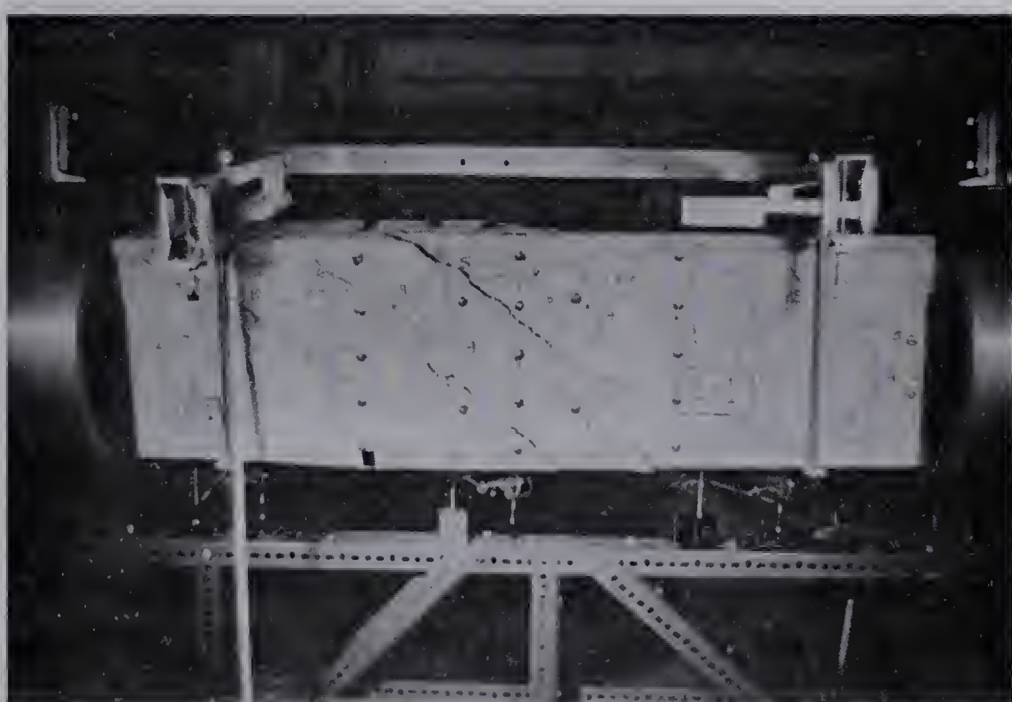


FIGURE 4–4 CRACK PATTERN FOR NORTH FACE OF BEAM E–1

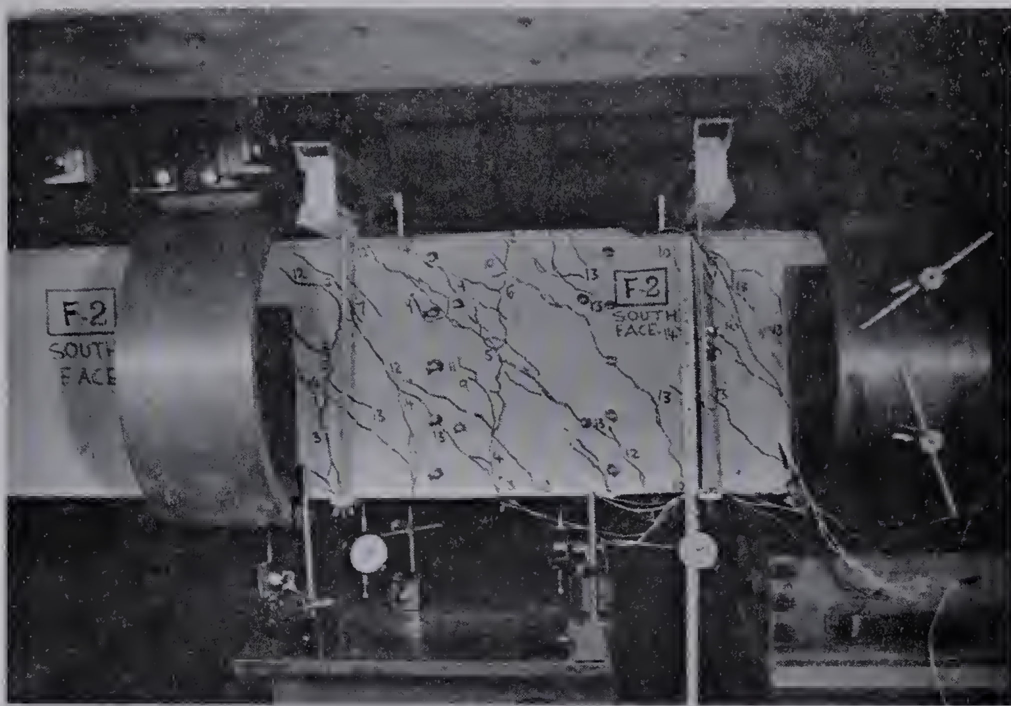


FIGURE 4–5 CRACK PATTERN FOR SOUTH FACE OF BEAM F–2

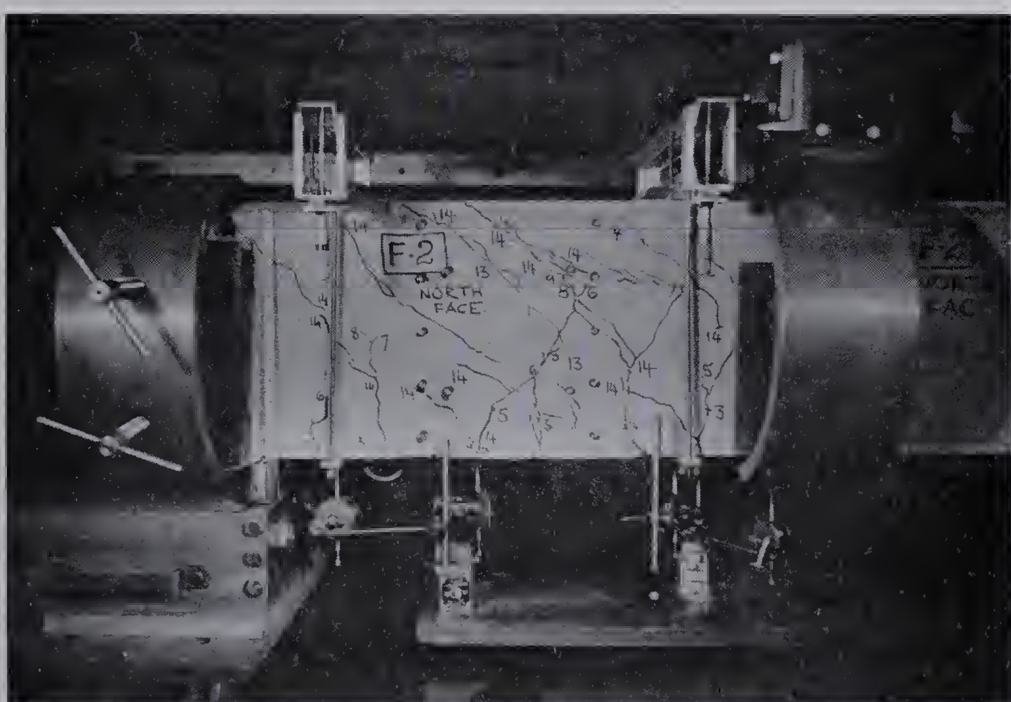


FIGURE 4–6 CRACK PATTERN FOR NORTH FACE OF BEAM F–2

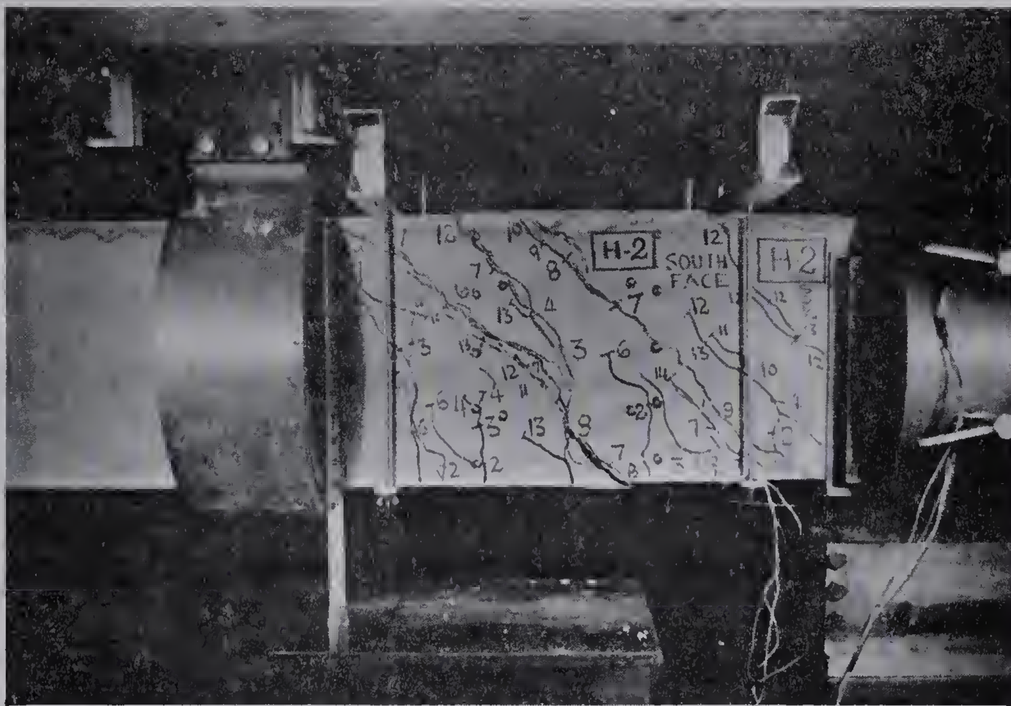


FIGURE 4–7 CRACK PATTERN FOR SOUTH FACE OF BEAM H–2

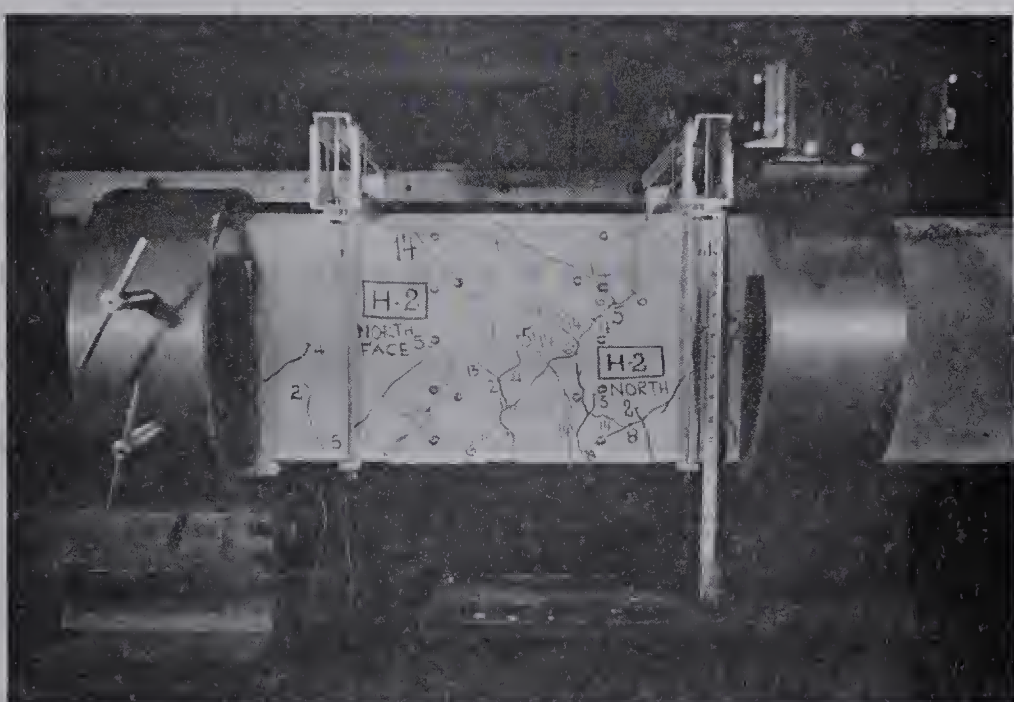


FIGURE 4–8 CRACK PATTERN FOR NORTH FACE OF BEAM H–2

shearing stresses. The transverse shear cracks and the torsional shear cracks crossed at about 90^{o} on the North face. The application of torsion tended to close the transverse shear cracks and produce other cracks normal to these cracks. The torsional cracks on the North face appeared at higher levels of torque than those on the South face. The sequence of cracking is shown in FIGURES 4-5 through 4-8 and TABLES B-25 through B-36.

4-4 Concrete Strains

The plots of the longitudinal and diagonal strains in concrete for all the beams are presented in FIGURES B-1 through B-11, Appendix B. The designations of the locations, where strains were measured, are shown in FIGURES 3-2 and 3-3. The strains were measured over 8-in. gage length using Demec mechanical strain gage.

(i) Combined Bending and Torsion

General trends of the concrete strains in beams subjected to bending and torsion are shown in FIGURE 4-9. The strains at Locations 6 and 7 on the vertical faces of the beam were negligible upto the commencement of twisting. As torque was applied, compressive and tensile strains were recorded at Locations 6 and 7 respectively. There were compressive strains at Locations 8 and 9 on the top face due to flexure. On application of torsion, the compressive strain at Location 8 increased whereas that at Location 9 decreased leading to strain reversal at higher torque. Near failure, the strain at Location 9 was tensile.

(ii) Combined Bending, Torsion and Shear

General trends of the concrete strains in beams subjected to bending, torsion and shear are shown in FIGURE 4-10. Due to the effect of

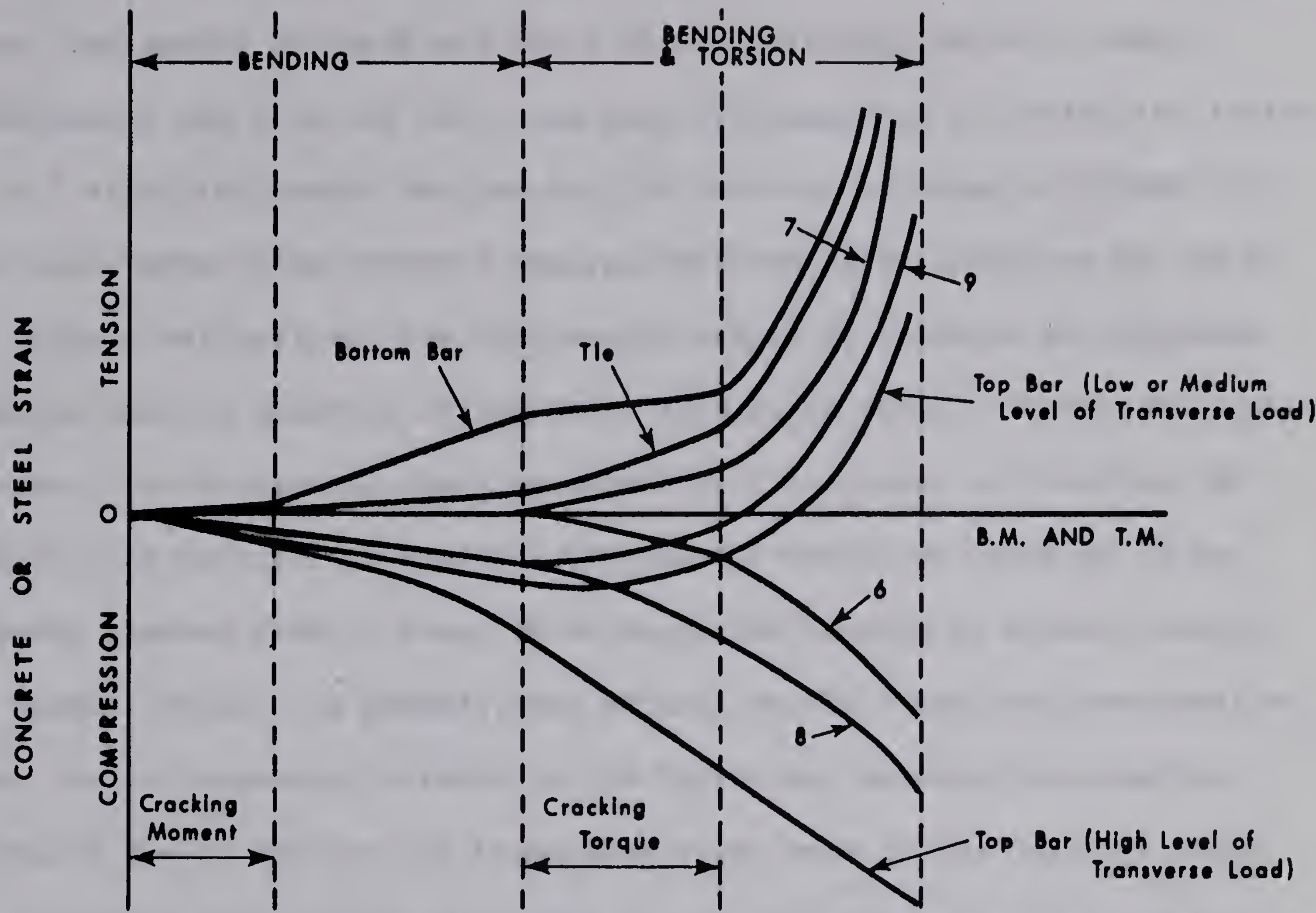


FIGURE 4-9 GENERAL TRENDS OF CONCRETE & STEEL STRAINS FOR BEAMS SUBJECTED TO COMBINED BENDING & TORSION

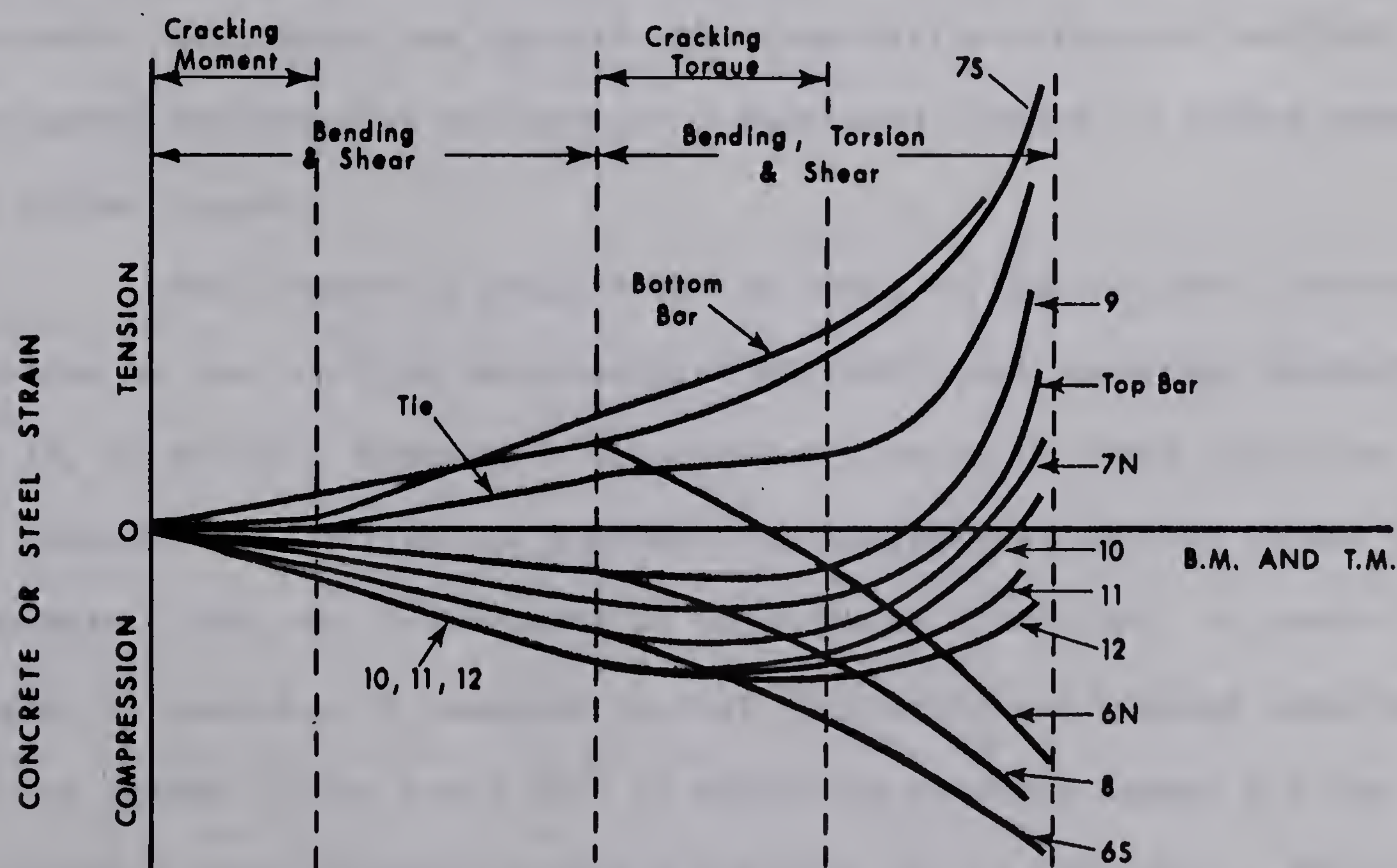


FIGURE 4-10 GENERAL TRENDS OF CONCRETE & STEEL STRAINS FOR BEAMS SUBJECTED TO COMBINED BENDING, TORSION & SHEAR

transverse shear, the concrete strains on the two vertical faces of the beam, designated as North and South faces, were not similar. Hence Locations 6 and 7 on the two sides were distinguished by adding the letter N or S after the number designating the location as shown in FIGURE 3-3. The transverse shear produced compressive strains at Locations 6S and 7N. As torsion was applied, the compressive strain at Location 6S increased whereas that at Location 7N decreased leading to strain reversal at higher torque. The transverse shear produced tensile strains at Locations 6N and 7S. As torsion was applied, the tensile strain at Location 7S increased whereas that at Location 6N decreased leading to strain reversal at higher torque. In general, the strains on the North face were smaller than the corresponding strains on the South face because the shearing stresses due to torsion and transverse shear were of the opposite sense on the North face whereas they were of the same sense on the South face of the beam. There were compressive strains at Locations 8 and 9 due to flexure. As torsion was applied, the compressive strain at Location 8 increased whereas that at Location 9 decreased leading to strain reversal at higher torque.

Since beams in group H had no steel on the top face, concrete strains on the top face were measured at additional locations designated as 10, 11 and 12. There were compressive strains at these locations due to flexure. As torsion was applied, the compressive strains tended to decrease. The rate of decrease of compressive strain was, in general, higher at Location 10 compared to that at Location 12 because Location 10 was nearer to the South face on which the twisting moment and the transverse shear produced shearing stresses of the same sign. Strain

reversals from compression to tension were recorded at Locations 10, 11 and 12 in Beam H-2 and at Locations 10 and 12 in Beam H-3 as shown in FIGURE B-9, Appendix B.

The longitudinal strains at Locations 1 through 5 were plotted along the depth of the beam and are presented in FIGURES B-10 and B-11, Appendix B. For the case of combined bending and torsion, the strains plotted are the average of those on the two sides of the beam. Since shear produced dissimilar conditions on the two sides, the strains on the North and South faces were plotted separately in the case of combined bending, torsion and shear. The strains were fairly linear before commencement of twisting but became non-linear as torsion increased.

4-5 Steel Strains

Plots of the steel strains for all reinforced concrete beams subjected to combined bending and torsion are given in FIGURES B-2 through B-5 and those for beams subjected to combined bending, torsion and shear are given in FIGURES B-7 through B-9,Appendix B.

(i) Combined Bending and Torsion

General trends of the steel strains in beams subjected to combined bending and torsion are illustrated in FIGURE 4-9. As the torsion was applied, the strain in the bottom steel increased slowly at first and at a higher rate when the cracking torque was exceeded. In most cases, the bottom steel reached the yield strain as the peak torque was approached. The strain in the tie was very small until twisting was commenced. The tensile strain in the tie increased abruptly after the cracking torque was exceeded. In most cases, the transverse

steel reached the yield strain at ultimate torque.

Before commencement of twisting, the compressive strain in the top longitudinal steel depended on the level of the transverse load. As torsion was applied, the compressive strain in the top longitudinal steel generally decreased unless the flexural compression was extremely high. The application of torsion reduced compressive strain in the top longitudinal steel in all cases except in the case of Beam D-1 which was subjected to the maximum flexural load. In the case of Beam D-1, the top steel yielded due to the flexural load. As torsion was applied, the compressive strain in the top steel continued to increase. Strain reversal from compression to tension was recorded for Beams C-2, C-3, D-2, D-3 and E-2. In case of Beam D-3, which had a small flexural load, the tensile strain in the top longitudinal steel at failure was close to the yield strain as shown in TABLE 4-1.

(ii) Combined Bending, Torsion and Shear

General trends for the steel strains in beams subjected to bending, torsion and shear are illustrated in FIGURE 4-10.

The strains in the bottom steel followed the same general trends as in case of bending and torsion. The transverse steel developed some tensile strain due to transverse shear which continued to increase as torsion was applied. In most cases the transverse steel attained yield strain at ultimate torque. The compressive strain in the top steel reduced on application of torsion. In some cases, such as Beams F-1, F-2, F-3, G-1, G-2 and G-3, there was strain reversal from compression to tension. In cases of small transverse load as for Beams F-3 and G-3, the top steel reached the yield strain in tension at the ultimate torque as shown in TABLE 4-2.

4-6 Deformation Characteristics

The deformation characteristics, comprising of the load-deflection and the torque-twist characteristics, for beams subjected to combined bending and torsion are shown in FIGURE B-12 and those for beams subjected to combined bending, torsion and shear are shown in FIGURE B-13. To study the effect of flexure on the torque-twist characteristics, the torque-twist curves were grouped and are shown in FIGURES 4-11 through 4-13.

(i) Load-Deflection Characteristics

Deflections were measured after every increment of the transverse load until the desired transverse load was attained. Deflections were measured at each end and at the center of the gage length for beams tested in combined bending and torsion as shown in FIGURE 3-2. The graphs of the deflection at the middle of the gage length relative to the ends of the gage length versus the transverse load are given in FIGURE B-12. For the beams tested in combined bending, torsion and shear, the deflections were measured at four points as indicated in FIGURE 3-3. The graphs of the deflections at these four points versus the transverse load are given in FIGURE B-13, Appendix B.

(ii) Torque-Twist Characteristics

The torque-twist curves are given separately for each beam in FIGURES B-12 and B-13, Appendix B and are grouped in FIGURES 4-11 through 4-13. In the case of plain concrete beams, flexure seemed to have no definite effect on the torque-twist curves. In the case of reinforced concrete beams, the presence of higher transverse load tended to reduce the initial slope of the torque-twist curve. This effect is discussed further in Section 6-7.

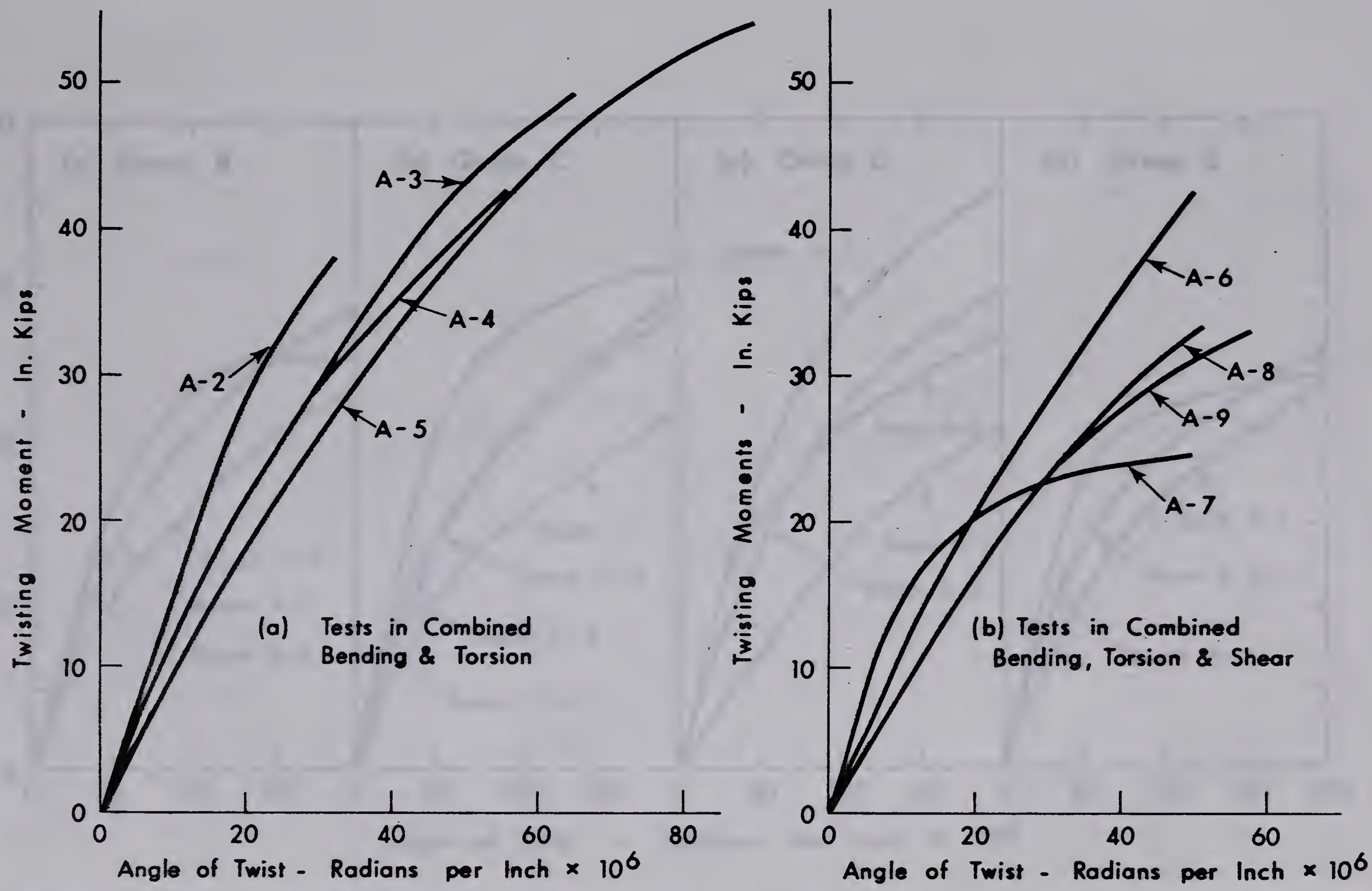


FIGURE 4-11 TORQUE-TWIST CHARACTERISTICS FOR BEAMS IN GROUP A

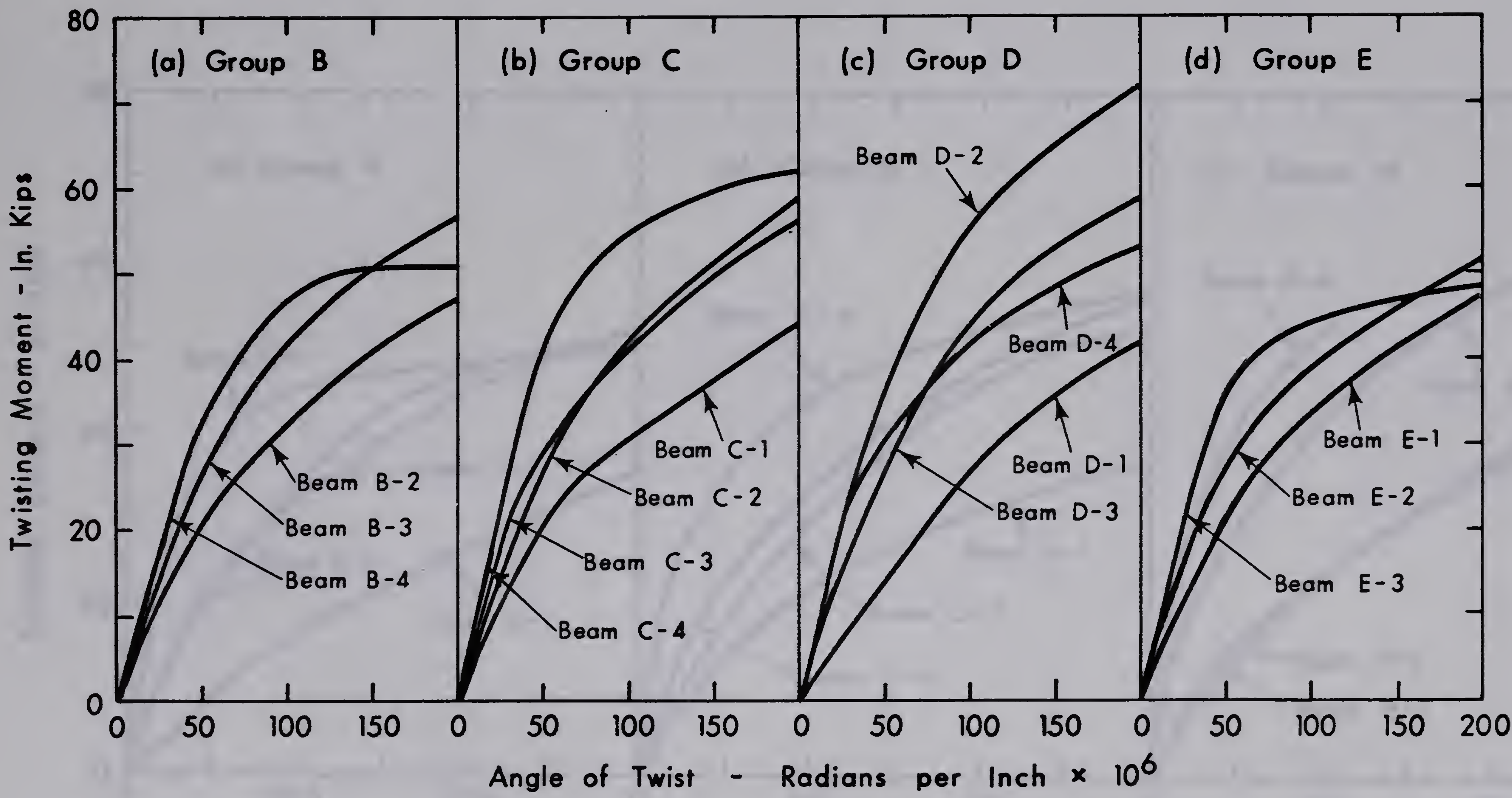


FIGURE 4-12 TORQUE - TWIST CHARACTERISTICS FOR BEAMS IN GROUPS B THROUGH E

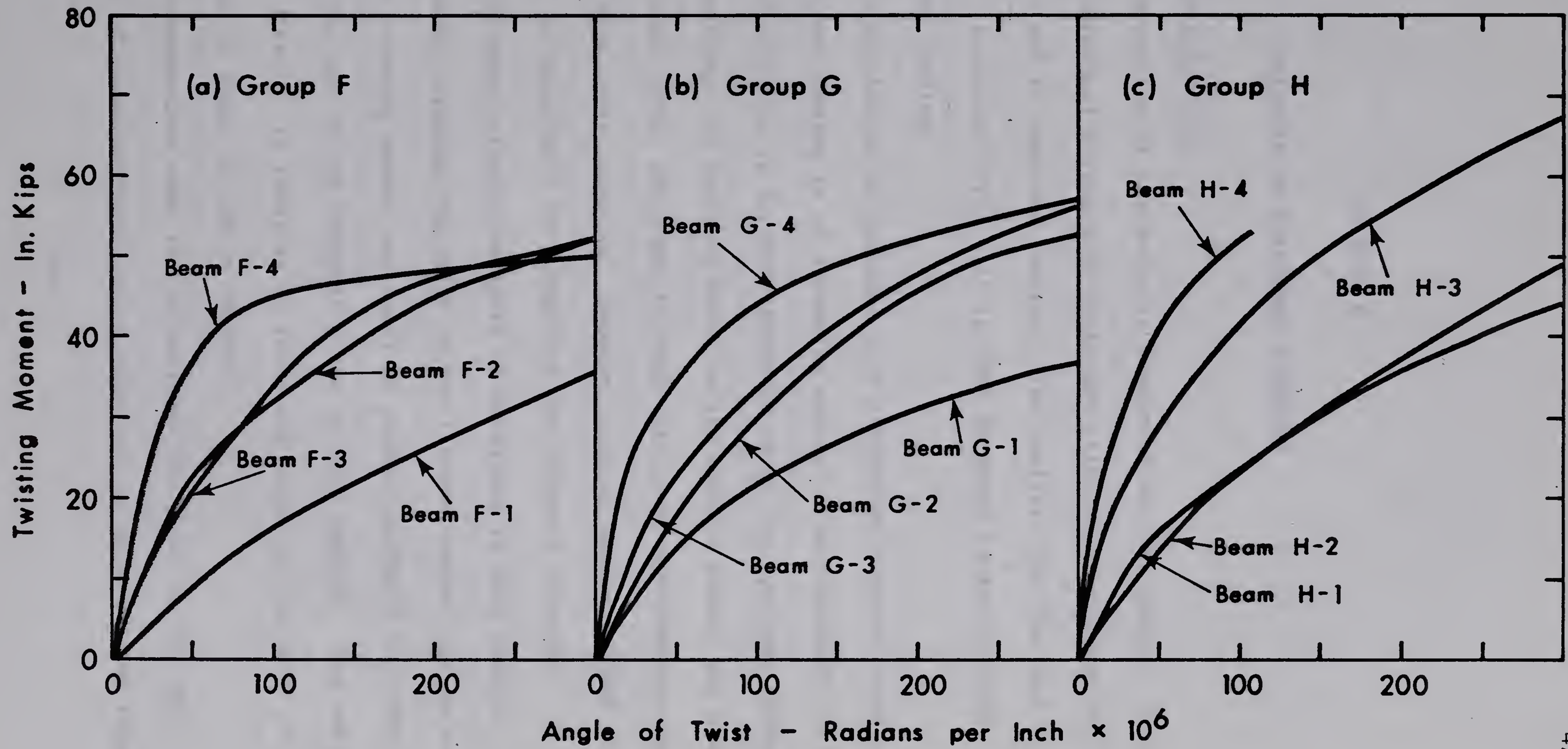


FIGURE 4-13 TORQUE - TWIST CHARACTERISTICS FOR BEAMS IN GROUPS F THROUGH H

CHAPTER V

ANALYSIS OF TORSIONAL STRENGTH

5-1 Introductory Remarks

The objective of the analysis presented in this chapter is to determine the torsional strength of plain and reinforced concrete sections in pure torsion and in combined loading. The analysis for the case of pure torsion is presented first. It is then extended to the general case of combined loading.

The distribution of stresses in a reinforced concrete beam subjected to combined loading is of a complex nature. The complexity of the distribution of stresses is further increased after the specimen suffers large scale cracking. An analysis based on the elastic theory has been presented by Cowan (1953) which may be used with some approximation upto the stage of large scale cracking. However, such an analysis based on the elastic theory is unsuitable for computing the ultimate strength.

An ultimate strength analysis is developed in this chapter. The analysis begins by assuming the value of the flexural moment acting on the beam. The twisting moment necessary to produce collapse can then be calculated by the use of the analysis given in this chapter. The analysis is simple and direct. It involves no trial and error if the level of the flexural moment is known or can be assumed.

The assumptions made in the analysis are given in the next section. Most of these assumptions are based on the observations reported

in CHAPTER IV. Others have been introduced for simplicity.

5-2 Assumptions of Analysis

The following assumptions have been made in the analysis:

(1) The torsional strength of a reinforced concrete section is the sum of the strength of the equivalent plain concrete section and the contribution of steel to the torsional strength.

(2) There is a redistribution of stresses before failure. The extent of this redistribution is indicated in Assumptions 5 and 7 below. It is discussed further in Section 7-5.

(3) The apparent unit torsional strength of concrete is determined according to Cowan's theory of failure under combined stresses presented in Appendix C.

(4) The stresses in any tie, which is intersected by a potential failure crack, is in the yield range at failure provided adequate longitudinal steel exists at the top and the bottom.

(5) The contribution of steel to the torsional strength is governed by the yielding of the top steel or the bottom steel or the transverse steel, whichever occurs first.

(6) The longitudinal bars give rise to lateral forces which contribute to the torsional strength. This contribution is limited to the torque produced by the maximum distributed lateral forces that the longitudinal bars can resist as cantilevers with a span equal to the spacing of the ties. The bars are assumed to develop their full plastic moments provided adequate lateral support is provided by the ties.

(7) The flexural compression is assumed to be uniformly distributed over

an area equal to one-fourth of the area of cross-section of the beam.

(8) The twisting moment reduces the lever arm, jd, of the internal flexural moment due to splitting of concrete on the top. The variation in the lever arm is assumed to be linear over the entire range from pure flexure to pure torsion.

The first assumption is the same as made by Cowan (1950) in his analysis of torsional strength of reinforced concrete beams. Ernst (1957) believed that at ultimate torque, the contribution of concrete recedes in importance. However, on account of the greater toughness and consequently greater redistribution of stresses in reinforced concrete sections, it can be assumed that the concrete continues to contribute a torque resistance equal to that of the equivalent plain concrete section.

The second assumption is supported by the extensive tests by Nylander (1945), Gardner (1960) and the present tests.

The third assumption deals with Cowan's theory of failure of concrete under combined stresses. This theory, which combines the Rankine's principal stress theory with the Coulomb's internal friction theory, was found to lead to satisfactory correlation with test results. The theory is simple and offers a physical interpretation of the mechanism of failure of concrete under combined stresses.

The fourth assumption is strongly supported by Chinenkov's tests and also by the test results presented in CHAPTER IV.

The fifth assumption is supported by test results presented in CHAPTER IV. This becomes evident when the torsional strengths of Beams G-4 and H-4 are compared. This is discussed further in Section 7-5.

The sixth assumption is based on the observation that for

identical transverse steel, an increase in longitudinal steel results in an increased ultimate torque. This is evident by comparison of the torsional strengths of Beams C-4 and D-4. The increase in torsional strength decreases as the tie spacing increases and for large tie spacing, this increase is negligible (Ernst, 1957).

Assumptions 7 and 8 have been introduced for simplicity. The stress in the compressed concrete is believed to become fairly uniform as failure is approached. Chinenkov (1959) stated that an exact determination of the depth of the compression zone at failure under combined loading is difficult because of the splitting of the concrete in the compression zone due to torsion. Gesund and Boston (1964) stated that from consideration of the statical equilibrium, it is likely that only about one-fourth of the cross-sectional area of a beam is in compression.

The last assumption deals with the reduction of lever arm due to torsion. Gesund and Boston (1964) have indicated the possibility of great reduction in lever arm due to torsion. Chinenkov (1959) gave sketches showing non-cracked concrete under the action of combined bending and torsion. The zone of flexural compression tends to shift from the top towards the center of the cross-section as the ratio of torsion to flexure varies from zero to infinity. The exact nature of the variation of lever arm is not known. The linear law is assumed for simplicity.

5-3 Pure Torsion

(i) Plain Concrete

The torsional strength of plain concrete is governed by the tensile strength of concrete. In the case of pure torsion, the unit

torsional strength of plain concrete τ_o may be taken equal to the tensile strength of concrete in direct tension as discussed in Section 6-2. Hence the torsional strength of a plain concrete section is given by Equations (5-1).

$$T_{co} = k_p b_o^2 d_o \tau_o = k_p b_o^2 d_o f_t' \qquad (5\text{-}1)$$

where T_{co} = torsional strength of a plain concrete section in pure torsion

k_p = coefficient for the fully plastic section in torsion

$= \frac{1}{2}\left(1 - \frac{1}{3}\frac{b_o}{d_o}\right)$

b_o = overall width of the rectangular section

d_o = overall depth of the rectangular section

τ_o = unit torsional strength of plain concrete in pure torsion

f_t' = tensile strength of concrete in direct tension

Since the form of the following equation based on elastic theory is identical to that of Equation (5-1), the same value of T_{co} may be obtained from two essentially different theories by proper choice of the value of τ_o.

$$T_{co} = k_e b_o^2 d_o \tau_o$$

where k_e = coefficient for the elastic section in torsion

(ii) <u>Reinforced Concrete</u>

The ultimate twisting moment for a reinforced concrete section subjected to pure torsion is given by the following equation.

$$T_{uo} = T_{co} + T_s \tag{5-2}$$

where T_{co} = torsional strength of plain concrete section in pure torsion as given by Equation (5-1),

and T_s = total torsional resistance of steel

The total torsional resistance of steel is given by Equation (5-3).

$$T_s = T_{s1} + T_{s2} \tag{5-3}$$

where T_{s1} and T_{s2} are the contributions of the transverse steel and the longitudinal steel respectively to the torsional strength.

The contribution of the transverse steel, T_{s1}, is determined from the number of ties intersected by a potential failure crack on the faces of the beam. The twisting moment produces a state of pure shear giving rise to diagonal tension at 45° to the axis of the beam. The diagonal tension has equal components in longitudinal and transverse directions. The ties intersected by a potential failure crack would reach the yield point at ultimate torque provided enough longitudinal steel exists both at the top face and at the bottom face to take up the longitudinal component of diagonal tension. If enough longitudinal steel does not exist either at the top or at the bottom, the ties cannot be stressed to the yield point and the contribution of transverse steel to torsional strength is correspondingly reduced.

Let n_1 = number of ties intersected by a potential failure crack on the longer side of the rectangle.

n_2 = number of ties intersected by a potential failure crack on the shorter side of the rectangle.

Then,

$$T_{s1} = n_1 a_v f_{st} b' + n_2 a_v f_{st} d' \tag{5-4}$$

where b' and d' = width and depth of the tie on the center lines of the legs

a_v = area of cross-section of one leg of the tie

f_{st} = stress in the transverse steel as given below

It is assumed that the ties would yield only if the yield loads of the top and bottom longitudinal bars are not less than the yield load of the ties. Hence,

$f_{st} = f_{yt}$, the yield stress of transverse steel if

$$\begin{aligned} &A_s . f_y \geqslant (n_1 + n_2)\ a_v f_{yt}\ , \text{ and} \\ &A'_s . f'_y \geqslant (n_1 + n_2)\ a_v f_{yt} \end{aligned} \tag{5-5}$$

where A_s = area of the bottom longitudinal steel

A'_s = area of the top longitudinal steel

f_y = yield stress of the bottom longitudinal steel

f'_y = yield stress of the top longitudinal steel

The above two inequalities ensure that sufficient longitudinal forces can be mobilised both at the top face and at the bottom face of the beam to develop the full yield stress in the transverse steel. However, if either one or both of the above inequalities are violated, f_{st} is linearly reduced. This is expressed by Equations (5-6).

$$f_{st} = f_{yt} \left[\frac{A_s f_y}{(n_1+n_2)\ a_v f_{yt}} \right]$$

or (5-6)

$$f_{st} = f_{yt} \left[\frac{A'_s\ f'_y}{(n_1+n_2)\ a_v f_{yt}} \right]$$

whichever is smaller. Equations (5-6) indicate that the stress developed in the transverse steel is governed by the yielding of either the bottom

steel or the top steel, whichever occurs first.

The twisting moment has a tendency to bend the longitudinal bars in the lateral direction, giving rise to lateral forces at each longitudinal bar. The contribution of longitudinal steel, T_{s2}, appears mainly to be a function of the size and position of longitudinal bars and the tie spacing. Tests (Ernst, 1957) have shown that for large tie spacing, the gain in torsional strength due to a higher proportion of longitudinal steel is negligible. Since the actual action of longitudinal bars is very complex, some idealisation is necessary. An idealised model, which assumes that the longitudinal bars act as cantilevers with a span equal to tie spacing, was found to give a close agreement with test results. In majority of cases, T_{s2} is small and may even be ignored in a conservative analysis of torsional strength.

The idealised action of longitudinal bars is illustrated in FIGURE 5-1. Referring to Assumption (6) of Section 5-2,

$$T_{s2} = \sum F.r \qquad (5\text{-}7)$$

where F = lateral force developed by a longitudinal bar

r = distance of the longitudinal bar from the axis of the beam.

The summation is taken for all the longitudinal bars. The lateral force, F, is given by Equation (5-8).

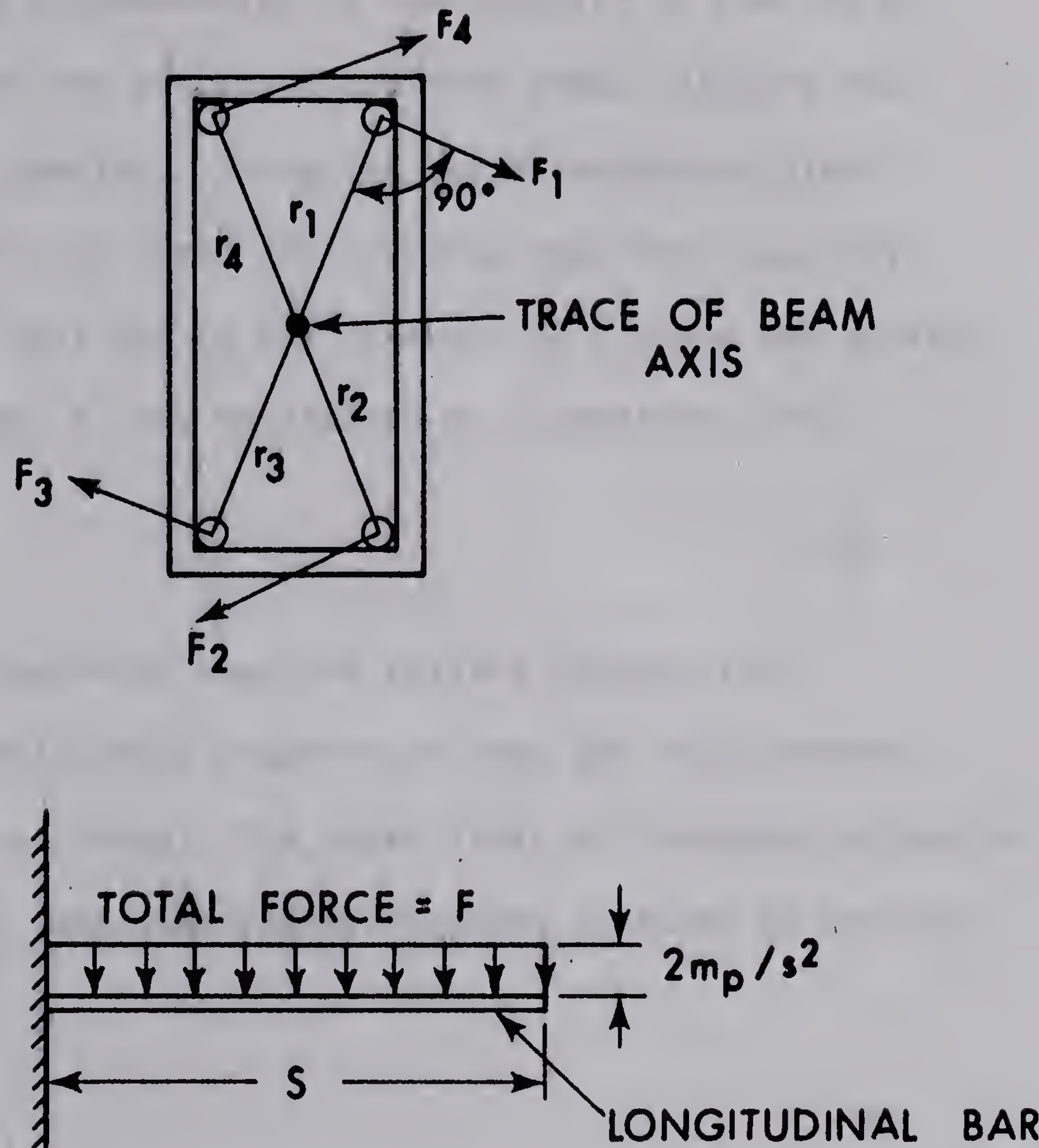
$$F = 2m_p/s \qquad (5\text{-}8)$$

where m_p = fully plastic moment of a longitudinal bar

= plastic section modulus of the bar times the yield stress of the bar material

s = spacing of the ties

The lateral force, F, as given by Equation (5-8), can be



$$T_{S2} = F_1 \cdot r_1 + F_2 \cdot r_2 + F_3 \cdot r_3 + F_4 \cdot r_4$$

$$= \sum F \cdot r \qquad (5-7)$$

FIGURE 5-1 CANTILEVER ACTION OF LONGITUDINAL BARS

developed only if the ties provide adequate lateral support to the longitudinal bar. The lateral force, F , induces shear force and probably some bending in the tie at failure. The tie also carries direct tension due to torsion. The determination of the capacity of the tie to provide lateral support under the action of combined shear, flexure and direct tension is extremely complex. Using the von Mises-Hencky Yield Criterion for yielding under pure shear and assuming the shear capacity of the tie to be reduced to half due to the presence of flexure and direct tension, the maximum value of F may be limited by Inequality (5-9).

$$F \leqslant \frac{a_v f_{yt}}{2\sqrt{3}} \qquad (5\text{-}9)$$

Ernst (1957) has indicated that the failure changes from diagonal tension type to hybrid shear (compression) type for large amounts of longitudinal and transverse steel. The upper limit of torsional strength, computed from Equation (5-2), may, therefore, be taken as given by Equation (5-10),

$$T_{uo} = k_p b_o^2 d_o \, v_c' \qquad (5\text{-}10)$$

where v_c' = unit transverse shearing strength of concrete

The true shearing strength of concrete is difficult to assess. A value equal to three times the unit torsional strength of plain concrete, τ_o , is considered somewhat conservative. If this value of v_c' is assumed, Equation (5-10) limits the torsional strength of a reinforced concrete section to three times the torsional strength of the equivalent plain concrete section. The tests in this investigation and those reported by Ernst (1957) have shown that the torsional strength of sections with fairly

large longitudinal and transverse steel was less than three times the torsional strength of equivalent plain concrete section.

5-4 Combined Bending and Torsion

(i) Plain Concrete

Pure bending produces merely a stress gradient in concrete. The net axial force on the whole concrete section is zero. Besides, the stresses due to flexure are small being limited in magnitude to the tensile strength of concrete. Theoretically, there is some gain in the torsional strength for the portion of concrete which is in flexural compression and some loss in the torsional strength for the portion of concrete which is in flexural tension. The above two effects tend to cancel each other. Besides, each effect is small since the flexural stresses in a plain concrete beam are small. Hence, the torsional strength of a plain concrete section is not significantly affected by the presence of flexural moment. This is in conformity with the test evidence in this investigation. Hence the torsional strength may be computed from Equation (5-1).

(ii) Reinforced Concrete

As in the case of pure torsion, the torsional strength of beams subjected to combined bending and torsion is assumed to be the sum of the torque resistances offered by the concrete and the steel. This is expressed by Equation (5-11).

$$T_u = T_c + T_s \tag{5-11}$$

where T_u = ultimate torque of a reinforced concrete section in combined loading

T_c = torsional strength of concrete in combined loading

T_s = total torsional resistance of steel

The torque resistance offered by concrete is the sum of the torques resisted by the concrete compressed due to flexure and that resisted by the non-compressed concrete.

$$T_c = T_{c1} + T_{c2} \qquad (5\text{-}12)$$

where T_{c1} = torsional strength of compressed concrete

T_{c2} = torsional strength of non-compressed concrete

An exact analysis of the torsional resistance of a reinforced concrete section with flexural cracks is not possible. Hence in the following analysis, an approximation has been introduced by using the modified sand heap analogy as indicated in FIGURES 5-2 and 5-3.

From mechanics of simple flexure,

Total tension = Total compression

$$A_s f_s = C = \frac{M_u}{jd} \qquad (5\text{-}13)$$

where M_u = ultimate bending moment in combined bending and torsion

jd = lever arm

C = total flexural compression

f_s = stress in bottom longitudinal steel due to flexure

The value of the lever arm, jd, is assumed to depend on the ratio:

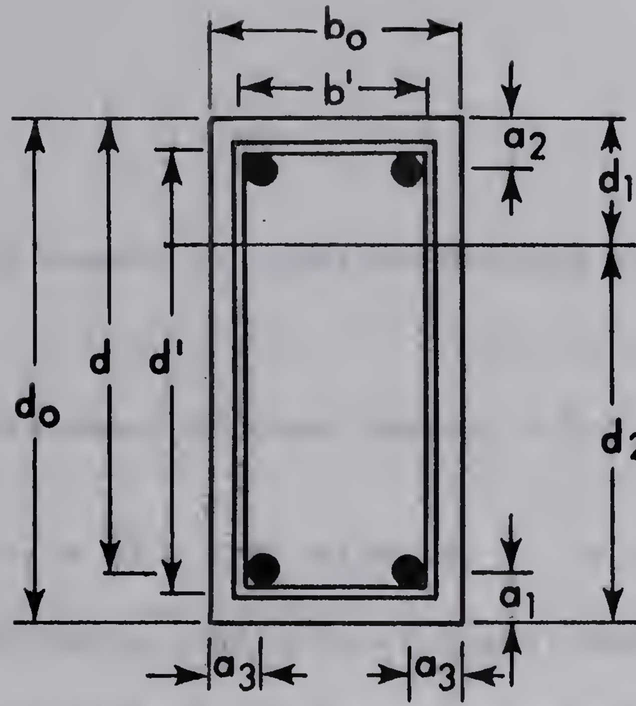


FIGURE 5-2 CROSS-SECTIONAL DIMENSIONS

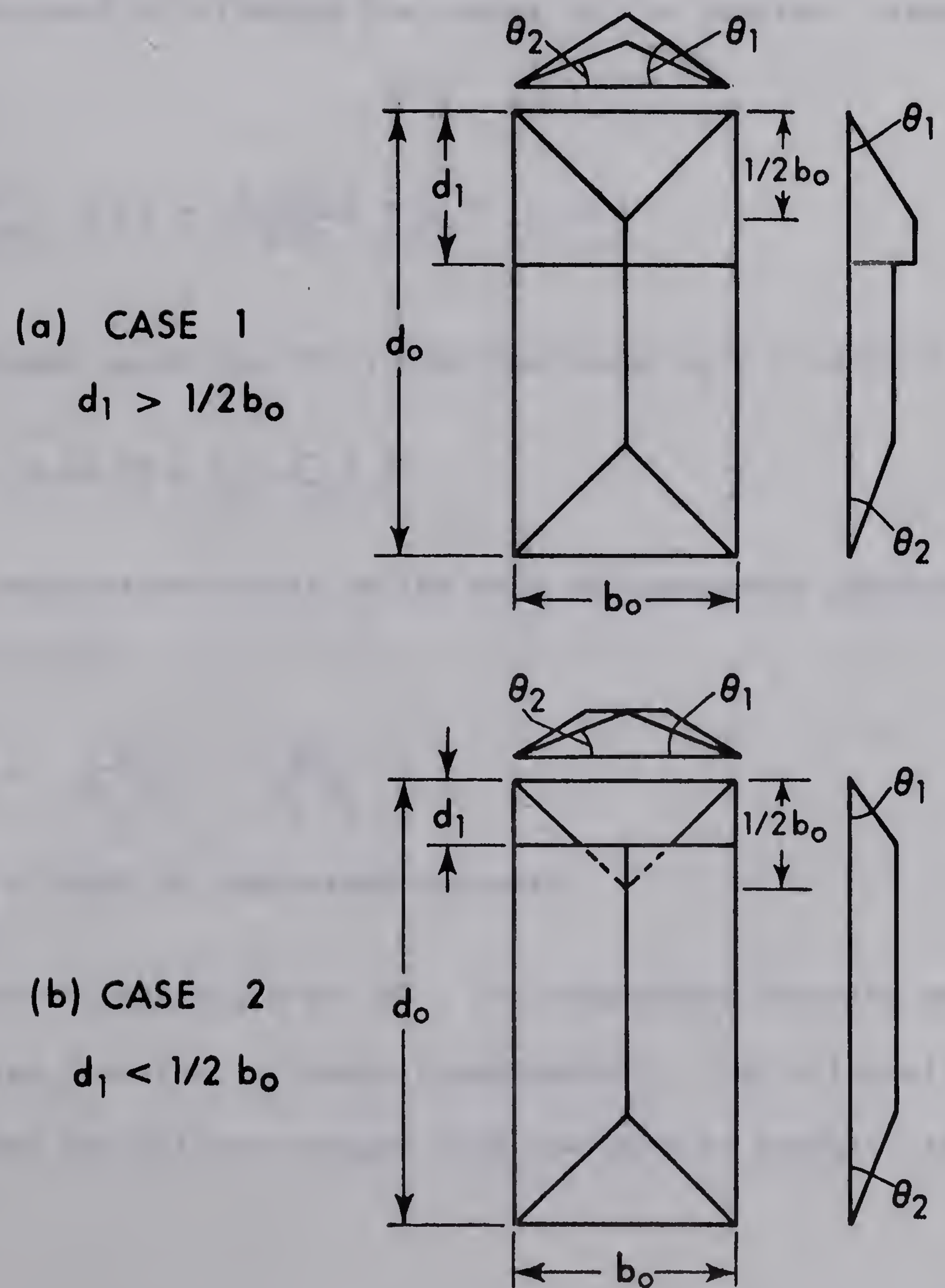


FIGURE 5-3 MODIFIED SAND HEAP ANALOGY

$$k_1 = \frac{M_u}{M_{uo}} \tag{5-14}$$

where M_u = ultimate bending moment in combined bending and torsion

M_{uo} = assumed ultimate moment in pure bending = 0.9 Asfy . d

For the case of pure bending (k_1 = 1) , the value of j is of the order of 0.9. As the value of k_1 decreases, the zone of compressed concrete shifts downwards due to the splitting of the concrete on the top. Hence the value of jd decreases. In the case of a very low value of k_1 , the compressed concrete is close to the center of the section. Assuming $d/d_o = 0.9$,

$$\lim_{k_1 \to 0} (j) = \frac{0.5-0.1}{0.9} = \frac{4}{9} \doteq 0.45 \tag{5-15}$$

Assuming a linear variation of j for the range $k_1 = 0$ to $k_1 = 1$,

$$j = 0.45\ (1 + k_1) \leqslant 0.9 \tag{5-16}$$

The average compressive stress in the zone of compressed concrete is given by Equations (5-17).

$$\sigma = \frac{C}{d_1\ b_o} = \frac{4C}{d_o\ b_o} \tag{5-17}$$

where d_1 = depth of compressed concrete.

Depending upon the magnitude of σ , the compressed concrete may fail due to cleavage (tension) or shear (compression). The critical value of σ , at which the failure changes from one type to another, is shown

in Appendix C as:

$$\sigma_{cr} = f'_c - \frac{2\,\tau_o}{1-\sin 37^o} \tag{5-18}$$

The apparent unit torsional strength of the compressed concrete may be determined by Equations (5-19) or (5-20).

For cleavage (tension) failure -

$$\tau_1 = \sqrt{(\sigma + \tau_o)\,\tau_o} \tag{5-19}$$

For shear (compression) failure -

$$\tau_1 = \sqrt{R^2 - (\sigma/2)^2} \tag{5-20}$$

where $R = \left[\frac{1}{2}\, f'_c\,(\text{cosec } 37^o - 1) + \frac{\sigma}{2}\right] \sin 37^o$

f'_c = compressive strength of concrete

The modified sand heap analogy for the case of combined bending and torsion, shown in FIGURE 5-3, is used for determining T_c. The analogy, as outlined below, is approximate. It involves a stress discontinuity at the junction of the compressed concrete and the non-compressed concrete. The sand heap is assumed to consist of two parts. The sand heap in the portion of the cross-sectional area, which corresponds to the zone of compressed concrete, is assumed to have a steeper slope than that of the sand heap over the remaining area. There are two cases as indicated below.

(a) Case 1 $d_1 \geqslant \frac{1}{2}\, b_o$

This case is shown in FIGURE 5-3(a). Taking the torque equal to twice the volume of the sand heap and replacing the slopes θ_1 and θ_2 by the analogous

unit torsional stresses τ_1 and τ_2 for the compressed and the non-compressed concrete respectively, Equation (5-21) is obtained.

$$T_c = \frac{1}{2} b_o^2 (d_1 - \frac{1}{6} b_o) \tau_1 + \frac{1}{2} b_o^2 (d_2 - \frac{1}{6} b_o) \tau_2 \qquad (5\text{-}21)$$

where d_1 and d_2 are the depths of the compressed concrete and the non-compressed concrete respectively.

(b) Case 2 $d_1 < \frac{1}{2} b_o$

This case is shown in FIGURE 5-3(b). Proceeding as in case 1, Equation (5-22) is obtained.

$$T_c = \left[\frac{1}{2} b_o^2 (d_1 - \frac{1}{6} b_o) + \frac{1}{12} (b_o - 2d_1)^3\right] \tau_1 + \left[\frac{1}{2} b_o^2 (d_2 - \frac{1}{6} b_o)\right] \tau_2 \qquad (5\text{-}22)$$

The torque resistance due to steel is given, as in the case of pure torsion, by Equation (5-3).

$$T_s = T_{s1} + T_{s2} \qquad (5\text{-}3)$$

where T_{s2} is given by Equations (5-7) through (5-9).

Equation (5-4) also holds

$$T_{s1} = n_1 a_v f_{st} b' + n_2 a_v f_{st} d' \qquad (5\text{-}4)$$

However, the capacity of the longitudinal bars to resist the longitudinal component of diagonal tension and diagonal compression due to torsion is affected on account of the presence of bending moment. Hence Inequalities (5-5) are modified to include the effect of flexure as indicated below.

$f_{st} = f_{yt}$ only if

$$A_s\ (f_y - f_s) \geqslant (n_1 + n_2)\ a_v f_{yt}\ ,\ \text{and}$$

$$A_s'\ (f_y' \pm f_s') \geqslant (n_1 + n_2)\ a_v f_{yt} \tag{5-23}$$

where f_s = stress in the bottom longitudinal steel due to flexure

f_s' = stress in the top longitudinal steel due to flexure

$= n\,\sigma$

n = modular ratio

In the second of Inequalities (5-23), the plus sign applies for cleavage (tension) failure and the minus sign for shear (compression) failure of the compressed concrete. The above two inequalities ensure that sufficient longitudinal forces can be mobilised both at the top face and at the bottom face of the beam to develop the full yield stress in the transverse steel. However, if either one or both of the above inequalities are violated,

$$f_{st} = f_{yt} \left[\frac{A_s\ (f_y - f_s)}{(n_1 + n_2)\ a_v f_{yt}} \right]$$

or (5-24)

$$f_{st} = f_{yt} \left[\frac{A_s'\ (f_y' \pm f_s')}{(n_1 + n_2)\ a_v f_{yt}} \right]$$

whichever is smaller. The plus sign in the latter equation applies in the case of cleavage (tension) failure and the minus sign applies in the case of shear (compression) failure of the compressed concrete. Equations (5-24) indicate that the stress developed in the transverse steel is governed by the yielding of the bottom steel or the top steel, whichever occurs first.

5-5 Combined Bending, Torsion and Shear

The shearing stresses due to transverse shear and torsion are of the same sign on one side of the beam and of the opposite sign on the other side of the beam. Nylander (1945) assumed that the strength of a beam in combined torsion and shear is obtained by equating the sum of the shear stresses due to torsion and transverse shear to the tensile strength of concrete.

$$\tau = \tau_o - v_u \tag{5-25}$$

where τ = apparent unit torsional strength in combined torsion and transverse shear

τ_o = unit torsional strength of plain concrete in pure torsion

= tensile strength of concrete in direct tension

v_u = nominal ultimate shear stress due to transverse shear in combined loading

The beams tested by Nylander had no transverse steel. The beams presumably had no ductility and the strength was governed by the most unfavorable stress combination.

In the case of combined bending, torsion and transverse shear,

$$\tau = \sqrt{\left(\frac{1}{2}\sigma + \tau_o\right)^2 - \left(\frac{1}{2}\sigma\right)^2} \pm v_u \tag{5-26}$$

The plus sign applies to the stress condition on the side of the beam where the shear stresses due to torsion and transverse shear have opposite sign and the minus sign applies to the stress condition on the other side of the beam. Equation (5-26) shows that the apparent unit torsional strength is increased on one side of the beam and decreased on the other

side. If sufficient transverse steel is provided to prevent a transverse shear failure, there will be redistribution of stresses tending to equalize the stresses on the two sides of the beam. If significant redistribution of stresses occurs, the torsional strength may not be appreciably reduced due to the presence of transverse shear.

CHAPTER VI

EVALUATION OF TEST RESULTS

6-1 Introduction

The analysis developed in the preceding chapter was used for predicting the torsional strength of 166 beams. Of these beams, 68 were tested by the author and 98 by other investigators as indicated below.

(i) Pure torsion

		No. of beams	Total
This investigation		9	
Andersen	(1935)	26	
Ernst	(1957)	18	
Author*	(1963)	30	
Author**	(1964)	4	87

(ii) Combined loading

		No. of beams	Total
This investigation		25	
Nylander	(1945)	16	
Cowan and Armstrong	(1955)	5	
Chinenkov	(1959)	13	
Gesund and Boston	(1964)	8	
Gesund, Schuette, Buchanan and Gray	(1964)	12	79
		Total	166

The analytical and the test results are presented in TABLES 6-2 through 6-12.

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* Pandit, G.S., "Torsion in Concrete Sections", M.Sc. Thesis, University of Alberta, 1963.

** Not published

6-2 Tensile Strength of Concrete

In most cases, the torsional strength of plain and reinforced concrete is governed by the tensile strength of concrete. However, the tensile strength of concrete is one of the most elusive properties of concrete. Various tests have been devised for determining the tensile strength of concrete but all these tests have certain shortcomings and limitations (Wright, 1955). Several investigators have presented empirical expressions relating the compressive strength, the tensile strength and the modulus of rupture. Due to variety of testing procedures and materials involved, these empirical relationships are often of limited application.

The tensile strength of concrete also appears to depend on the stress condition. McHenry and Karni (1958), on the basis of their tests on hollow cylinders subjected to internal fluid pressure and axial compression, stated that the tensile strength is reduced by an orthogonal compressive stress. These investigators found the relationship between stresses at failure to be nonlinear. Gardner (1960) stated that the direct compressive stress has two opposing effects: (i) it reduces the principal tensile stress, which occurs as a result of shear stress, and (ii) it may reduce the ability of concrete to withstand this tensile stress.

A number of empirical relationships connecting the compressive strength and the tensile strength of concrete have been suggested. Most of them are of the following form.

$$f_t' = k\ (f_c')^m \tag{6-1}$$

in which k and m are constants. Cowan and Armstrong (1955) used the following relationship.

$$f_t' = 1.2\ (\text{cube crushing strength})^{2/3} \tag{6-2}$$

The European Concrete Committee proposed the following equation.

$$\text{Modulus of rupture} = 9.5\ (f_c')^{0.5} \tag{6-3}$$

Gonnerman and Shuman (1928) presented Equation (6-4)

$$f_t' = 0.68\ (f_c')^{0.75} \tag{6-4}$$

Sozen, Zwoyer and Siess (1959) suggested the following empirical relationship.

$$\text{Modulus of rupture} = \frac{3{,}000}{4 + \dfrac{12{,}000}{f_c'}} \tag{6-5}$$

Wright (1955) found that indirect tension tests (Carneiro, 1947) on 6 x 12-in. cylinders give values of tensile strength of the order of 3/2 times those obtained by direct tension tests and of the order of 2/3 of the values of modulus of rupture of 4-in. beams tested under third-point loading. He also found that cylinders tend to give more uniform results than the other types of tensile test, but give less uniform results than the compression test. Wright believed that on account of the favourable redistribution of stress, the tensile strength, as determined by flexure tests on beams and indirect tests on cylinders, is greater than the true tensile strength of concrete.

On account of the greater uniformity of results of the compression tests and the indirect tension tests, control cylinders were utilized for the above two types of tests. The unit torsional strength of plain concrete was taken as 2/3 of the tensile strength of concrete based primarily

on the indirect tension tests on 6 x 12-in. cylinders. This implied, in accordance with Wright's findings, that the unit torsional strength of plain concrete was taken equal to the direct tensile strength of concrete. The principal tensile stress due to torsion is accompanied by an orthogonal compressive stress. The findings of McHenry and Karni (1958) would, therefore, seem to suggest a reduction in the value of unit torsional strength of plain concrete assumed above. Since no generalized criterion involving a reduction in the tensile strength due to the existence of orthogonal compression was developed by McHenry and Karni, and also because the possible reduction was considered to be small, the unit torsional strength of plain concrete was taken equal to the strength in direct tension without any reduction.

$$\tau_o = f_t' = k\ (f_c')^m \tag{6-6}$$

In order to utilize the results of compression tests on 6 x 12-in. cylinders, the tensile strength of concrete was also calculated from the compressive strength. Using Wright's conclusion that the tensile strength, as determined by indirect test on 6 x 12-in. cylinders, is 2/3 of the modulus of rupture, Equation (6-3) gives the following relationship.

$$f_t'\ \text{(indirect test)} = 6.33\ (f_c')^{0.5} \tag{6-7}$$

and the formula used by Gesund and Boston (1964) gives Equation (6-8)

$$f_t'\ \text{(indirect test)} = 5.33\ (f_c')^{0.5} \tag{6-8}$$

The author in his previous investigation (Pandit, 1963) found Equation (6-9) most suitable.

$$f_t' \text{ (indirect test)} = 5.7\ (f_c')^{0.5} \tag{6-9}$$

Since the value given by Equation (6-9) lies intermediate between those given by Equations (6-7) and (6-8) and also due to the similarity of materials and mix design used by the author in the present and the previous investigations, Equation (6-9) was used. The values of the tensile strength thus obtained and those determined directly from the indirect tension tests on 6 x 12-in. cylinders were averaged. The unit torsional strength of plain concrete was taken as 2/3 of this average value as shown in TABLE 6-1.

The same procedure as outlined above was also used for determining the unit torsional strength of plain concrete for beams tested by the Author in his previous investigations. The values of τ_o thus obtained are given in TABLES 6-5 and 6-6.

For calculating the value of τ_o for beams tested by other investigators, Equations (6-6) were used.

$$\tau_o = f_t' = k\ (f_c')^m \tag{6-6}$$

The value of m was taken equal to 0.5 in all cases. The value of k was taken equal to 3.8 unless there was adequate evidence to justify another value. For beams tested by Andersen (1935), a value of k equal to 4.4 was calculated from his tests on six beams of Group R using the formula for the fully plastic circular section. This value was used in TABLE 6-3. Similarly, Ernst's (1957) tests on three beams without ties gave a value of k equal to 3.13. This value was used in TABLE 6-4.

TABLE 6-1

UNIT TORSIONAL STRENGTH OF PLAIN CONCRETE

Beam	f'_c psi	f'_t (indirect test) psi	τ_o psi
A-1	4340	433	270
A-2	4790	460	285
A-3	4500	469	284
A-4	5200	460	291
A-5	5200	460	291
B-1	4560	467	284
B-2	4640	406	265
B-3	4690	397	263
B-4	5070	462	289
C-1	4980	442	281
C-2	4820	427	275
C-3	5340	502	307
C-4	5800	497	311
D-1	4880	449	283
D-2	5100	441	283
D-3	4650	441	277
D-4	5080	416	275
E-1	4910	381	260
E-2	5100	438	282
E-3	4700	422	271
A-6	5200	460	291
A-7	4850	418	272
A-8	3740	367	239
A-9	3750	306	219
F-1 and F-2	5060	384	263
F-3 and F-4	4650	370	253
G-1 and G-2	4560	357	247
G-3 and G-4	4920	348	249
H-1 and H-2	4450	352	244
H-3 and H-4	4830	350	249

Nylander (1945), Cowan and Armstrong (1955) and Chinenkov (1959) determined compressive strength of concrete from tests on cubes. The ratio of the compressive strengths of 6 x 12-in. cylinders and 6-in. cubes cast from the same batch of concrete was found by Cowan and Armstrong (1955) to be 0.95. Using this result, the value of τ_o is given by Equation (6-10).

$$\tau_o = f'_t = 3.8\ (0.95\ C_u)^{0.5} \tag{6-10}$$

where C_u = compressive strength of concrete determined from 6-in. control cubes.

Equations (6-10) were used for calculating the values of τ_o in TABLES 6-8 through 6-10.

6-3 Correlation of Tests in Pure Torsion

The analytical and test results for 87 beams tested in pure torsion are given in TABLES 6-2 through 6-6. Forty-three beams were tested by the author, 9 in this investigation and 34 in his previous investigations. Forty-four beams included here were tested by Andersen (1935) and Ernst (1957). All data necessary for evaluating the torsional strength of beams are given in TABLES 6-2 through 6-6.

The number of ties cut by a potential failure crack was based on the assumption that the torsion cracks were inclined at 45° to the axis of the beam. However, since the crack can deviate slightly, n_1 and n_2 were calculated using Equations (6-11).

$$n_1 = \frac{d_o - a_1 - a_2}{s}$$
$$n_2 = \frac{b_o - 2a_3}{s} \tag{6-11}$$

Since the potential failure crack is likely to be the one which intersects the minimum number of ties and since n_1 and n_2 must be integers, the right sides of Equations (6-11) were rounded off to the nearest smaller whole numbers. For example, if Equations (6-11) gave $n_1 = 2.7$ and $n_2 = 1.3$, the values of n_1 and n_2 were taken as 2 and 1 respectively. The symbols for the cross-sectional dimensions used in Equations (6-11) are explained in FIGURE 5-2.

TABLE 6-2

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN PURE TORSION IN THIS INVESTIGATION

Beam	Actual Size		Concrete Strength		Longitudinal Bars**		Ties**		b'	d'	T_{co}	T_{s1}	T_{s2}	T_{uo}	
	b_o in	d_o in	f'_c psi	τ_o* psi	At Top	At Bottom	Size	Spacing in	in	in				Analytical	Test
											inch - kips				
A-5	6.10	12.20	5200	291	Nil	Nil	Nil	-	---	--	55	0	0	55	58
B-4	6.10	12.20	5070	289	2-#3	2-#4	#3	6.0	4.63	10.63	55	29	5	89	85
C-4	6.50	12.20	5800	311	2-#3	2-#5	#3	4.5	4.63	10.63	66	58	11	135	111
D-4	6.20	12.20	5080	275	2-#5	4-#6	#3	4.5	4.63	10.63	53	58	37	148	146
E-3	6.20	12.20	4700	271	2-#4	2-#4	#3	4.5	4.63	10.63	53	58	11	122	121
A-9	6.20	12.20	3750	219	Nil	Nil	Nil	-	--	--	42	0	0	42	42
F-4	6.10	12.20	4650	253	3-#3	3-#3	#3	8.0	4.63	10.63	48	29	4	81	95
G-4	6.20	12.20	4920	249	3-#3	3-#3	#2	3.5	4.75	10.75	48	55	9	112	115
H-4	6.25	12.20	4830	249	Nil	2-#3 and 1-#6	#2	3.5	4.75	10.75	49	0	12	61	64

* From TABLE 6-1
** Strength properties of reinforcing steel are given in TABLE A-5, Appendix A

[illegible]

TABLE 6-3

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN PURE TORSION BY ANDERSEN (1935)

Beam	Actual Size		Concrete Strength		Longitudinal Bars**		Ties**		b'	d'	T_{co}	T_{s1}	T_{s2}	T_{uo}	
	b_o in	d_o in	f'_c psi	τ_o* psi	At Top	At Bottom	Size in	Spacing in	in	in	inch - kips			Analytical	Test
Group B_1															
1	10	10	2100	202	2-#4	2-#4	Nil	-	-	-	67	0	0	67	70
2	10	10	2250	209	2-#4	2-#4	Nil	-	-	-	70	0	0	70	77
3	10	10	2250	209	2-#4	2-#4	Nil	-	-	-	70	0	0	70	80
4	10	10	3600	264	2-#4	2-#4	Nil	-	-	-	88	0	0	88	85
5	10	10	3600	264	2-#4	2-#4	Nil	-	-	-	88	0	0	88	89
6	10	10	3680	267	2-#4	2-#4	Nil	-	-	-	89	0	0	89	98
7	10	10	5000	311	2-#4	2-#4	Nil	-	-	-	104	0	0	104	105
8	10	10	5000	311	2-#4	2-#4	Nil	-	-	-	104	0	0	104	109
9	10	10	5200	317	2-#4	2-#4	Nil	-	-	-	106	0	0	106	120
Group B_2															
1	10	10	1900	192	2-#4	2-#4	5/32	6	8	8	64	13	5	82	83
2	10	10	2100	202	2-#4	2-#4	5/32	6	8	8	67	13	5	85	85
3	10	10	2200	206	2-#4	2-#4	5/32	6	8	8	69	13	5	87	96
4	10	10	2950	239	2-#4	2-#4	5/32	6	8	8	80	13	5	98	101
5	10	10	3100	245	2-#4	2-#4	5/32	6	8	8	82	13	5	100	96
6	10	10	3400	257	2-#4	2-#4	5/32	6	8	8	86	13	5	104	102
7	10	10	5080	314	2-#4	2-#4	5/32	6	8	8	105	13	5	123	115
8	10	10	5300	320	2-#4	2-#4	5/32	6	8	8	107	13	5	125	112
9	10	10	5500	326	2-#4	2-#4	5/32	6	8	8	109	13	5	127	127

* Using m = 4.4 in Equations (6-6) as discussed in Section 6-2

** Yield stress: #4 bars 78.8 ksi, 5/32-in. tie 43.3 ksi

TABLE 6-3
(Continued)

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN PURE TORSION BY ANDERSEN (1935)

Beam	Actual Size		Concrete Strength		Longitudinal Bars**		Ties**		b'	d'	T_{co}	T_{s1}	T_{s2}	T_{uo}	
	b_o in	d_o in	f' psi	T_o^* psi	At Top	At Bottom	Size in	Spacing in	in	in	inch-kips			Analytical	Test
Group B_3															
1	10	10	2100	202	2-#4	2-#4	5/32	3	8	8	67	26	5	98	91
2	10	10	2200	206	2-#4	2-#4	5/32	3	8	8	69	26	5	100	94
3	10	10	2820	234	2-#4	2-#4	5/32	3	8	8	78	26	5	109	102
4	10	10	3480	260	2-#4	2-#4	5/32	3	8	8	87	26	5	118	109
5	10	10	3480	260	2-#4	2-#4	5/32	3	8	8	87	26	5	118	112
6	10	10	5050	313	2-#4	2-#4	5/32	3	8	8	104	26	5	135	119
7	10	10	5200	317	2-#4	2-#4	5/32	3	8	8	106	26	5	137	135
8	10	10	5250	319	2-#4	2-#4	5/32	3	8	8	106	26	5	137	130

* Using m = 4.4 in Equations (6-6) as discussed in Section 6-2
** Yield stress: #4 bars 78.8 ksi, 5/32-in.tie 43.3 ksi

TABLE 6-4

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN PURE TORSION BY ERNST (1957)

Beam	Actual Size		Concrete Strength		Longitudinal Bars***		Ties***		b'	d'	T_{co}	T_{s1}	T_{s2}	T_{uo}	
	b_o	d_o	f'_c*	τ_o**	At	At	Size	Spacing	in	in				Analytical	Test
	in	in	psi	psi	Top	Bottom		in			inch - kips				
3TR-0	6.0	12.0	3923	196	2-#3	2-#3	Nil	--	-	-	35	0	0	35	38
3TR-1	6.0	12.0	3923	196	2-#3	2-#3	#2	28	4.5	10.0	35	0	1	36	35
3TR-3	6.0	12.0	3923	196	2-#3	2-#3	#2	14	4.5	10.0	35	0	1	36	34
3TR-7	6.0	12.0	3923	196	2-#3	2-#3	#2	7	4.5	10.0	35	12	3	50	50
3TR-15	6.0	12.0	3923	196	2-#3	2-#3	#2	4	4.5	10.0	35	24	5	64	62
3TR-30	6.0	12.0	3923	196	2-#3	2-#3	#2	Double at 4	4.5	10.0	35	49	5	89	76
4TR-0	6.0	12.0	3923	196	2-#4	2-#4	Nil	--	-	-	35	0	0	35	34
4TR-1	6.0	12.0	3923	196	2-#4	2-#4	#2	28	4.5	10.0	35	0	1	36	32
4TR-3	6.0	12.0	3923	196	2-#4	2-#4	#2	14	4.5	10.0	35	0	2	37	35
4TR-7	6.0	12.0	3923	196	2-#4	2-#4	#2	7	4.5	10.0	35	12	5	52	55
4TR-15	6.0	12.0	3923	196	2-#4	2-#4	#2	4	4.5	10.0	35	24	9	68	74
4TR-30	6.0	12.0	3923	196	2-#4	2-#4	#2	Double at 4	4.5	10.0	35	49	9	93	85
5TR-0	6.0	12.0	3923	196	2-#5	2-#5	Nil	--	-	-	35	0	0	35	34
5TR-1	6.0	12.0	3923	196	2-#5	2-#5	#2	28	4.5	10.0	35	0	3	38	33
5TR-3	6.0	12.0	3923	196	2-#5	2-#5	#2	14	4.5	10.0	35	0	6	41	43
5TR-7	6.0	12.0	3923	196	2-#5	2-#5	#2	7	4.5	10.0	35	12	11	58	60
5TR-15	6.0	12.0	3923	196	2-#5	2-#5	#2	4	4.5	10.0	35	24	16	75	77
5TR-30	6.0	12.0	3923	196	2-#5	2-#5	#2	Double at 4	4.5	10.0	35	49	16	100	93

*Average of three 6 x 12-in. cylinders

**Using m = 3.13 in Equations (6-6) as discussed in Section 6-2

***Yield stress: #2 bars 55.5 ksi, #3 bars 53.6 ksi, #4 bars 41.0 ksi, and #5 bars 48.6 ksi

TABLE 6-5

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN PURE TORSION BY AUTHOR (1963)

Beam	Actual Size		Concrete Strength		Longitudinal Bars*		Ties*		b' in	d' in	T_{co}	T_{s1}	T_{s2}	T_{uo}	
	b_o in	d_o in	f'_c	τ_o	At Top	At Bottom	Size in	Spacing in			inch - kips			Analytical	Test
A.1, A.2 and A.3	4	4	3370	219	Nil	Nil	Nil	--	-	-	4.7	-	-	4.7	5.0, 5.9 and 6.6
B.1 and B.2	4	4	3560	230	2-#2	2-#2	Nil	--	-	-	4.9	0	0	4.9	7.0 and 5.5
C.1 and C.2	4	4	3560	230	2-#2	2-#2	0.144	6	2.5	2.5	4.9	0	0.2	5.1	6.8 and 6.3
D.1 and D.2	4	4	4060	237	2-#2	2-#2	0.144	3	2.5	2.5	5.0	0	0.5	5.5	7.0 and 6.5
E.1 and E.2	4	4	4060	237	2-#2	2-#2	0.144	3	2.5	2.5	5.0	0	0.5	5.5	6.0 and 6.5
F.1 and F.2	4	4	3710	242	2-#2	2-#2	0.144	2	2.5	2.5	5.2	2.0	0.7	7.9	7.2 and 7.0
G.1 and G.2	4	4	3710	242	2-#2	2-#2	0.144	Double at 2	2.5	2.5	5.2	4.0	0.7	9.9	7.0 and 7.5
H.1 and H.2	4	4	3335	221	3-#2	4-#2**	Nil	--	-	-	4.7	0	0	4.7	7.1 and 6.0
I.1 and I.2	4	4	3335	221	3-#2	4-#2**	0.144	2	2.5	2.5	4.7	2.0	1.3	8.0	7.6 and 7.6
J.1 and J.2	4	4	4190	245	2-#2	2-#2***	0.144	2	2.5	2.5	5.2	2.0	0.7	7.9	7.0 and 7.1
K.1 and K.2	4	4	4190	245	2-#3	2-#3	0.144	2	2.5	2.5	5.2	2.0	0.6	7.8	7.6 and 7.0
L.1 and L.2	4	4	4105	244	2-#3	2-#3	0.250	6	2.5	2.5	5.2	0	0.9	6.1	7.1 and 7.0
M.1 and M.2	4	4	4105	244	2-#3	2-#3	0.250	Double at 2-1/4	2.5	2.5	5.2	0	2.4	7.6	9.2 and 9.1
N.1, N.2 and N.3	4	4	3910	235	2-#3	2-#3	0.250	4	2.5	2.5	5.0	0	1.4	6.4	8.6, 8.1 and 9.8

*Yield stress: #3 bars 58.9 ksi, #2 bars 47.5 ksi and 0.144-in wire 24.1 ksi

**These beams had in addition two #2 bars, one on the middle of each side

*** These beams had in addition five #2 bars, one coinciding with the beam axis and four arranged symmetrically at the corners of a 1-in. square

TABLE 6-6

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN PURE TORSION BY AUTHOR (1964)**

Beam	Actual Size		Concrete Strength		Longitudinal Bars*		Ties*		b' in	d' in	T_{co}	T_{s1}	T_{s2}	T_{uo}	
	b_o in	d_o in	f'_c	τ_o	At Top	At Bottom	Size	Spacing in						Analytical	Test
											inch - kips				
1	6.0	9.1	4500	253	Nil	Nil	Nil	-	-	-	32	0	0	32	35
2	6.0	9.1	4160	253	2-#4	2-#4	#2	2.0	4.25	7.25	32	44	9	85	79
3	6.0	9.1	4050	253	2-#4	2-#4	#3	4.5	4.13	7.13	32	26	8	66	67
4	6.0	9.1	5020	283	2-#2	2-#2 and 3-#4	#3	4.5	4.13	7.13	36	18	4	58	49

* Strength properties of reinforcing steel are given in TABLE A-5, Appendix A
** Not published

6-4 Correlation of Tests in Combined Bending and Torsion

The analytical and test results for 67 beams tested in combined bending and torsion are given in TABLES 6-7 through 6-12. Thirteen beams were tested by the author and 54 by other investigators. All data necessary for evaluating the torsional strength of beams subjected to combined bending and torsion using the analysis presented in the preceding chapter are given in TABLES 6-7 through 6-12.

The number of ties intersected by a potential failure crack was determined from Equations (6-11). The inclination of torsion cracks with the beam axis in the zone in which the concrete was not compressed by flexure was observed to be close to 45^{o} as can be seen in FIGURES 4-1 through 4-4. The inclination of torsion cracks with the beam axis in the zone compressed by flexure was less than 45^{o}. However, the value of n_2 is governed by the number of ties intersected by a torsion crack on the bottom face. Since concrete near the bottom face does not carry flexural compression, the use of Equations (6-11) is justified. Chinenkov (1959) also assumed a constant crack angle on all sides of the beam in his computations.

If a beam, which has a high percentage of tension steel and a low concrete strength, is subjected to large flexural load, it is possible that the computed value of the normal stress, σ , based on the assumption that the area of the compressed concrete is one-fourth of the total cross-sectional area, may be greater than f_c' . This was the case for the first five beams in TABLE 6-10. Since σ cannot exceed f_c' , it was taken equal to f_c' , and the area of the compressed concrete was correspondingly increased to develop the required flexural compression.

In the following tables, the analytical values of ultimate torque are given for $k_2 = 0$, 1/2 and 1 where k_2 is the ratio given by Equation (6-12).

$$k_2 = \frac{\tau_2}{\tau_o} \qquad (6\text{-}12)$$

For the tests in this investigation, the analytical values of the ultimate torque are also given for $k_2 = 3/4$ since this value of k_2 gave a better agreement between the analysis and the test results. Assuming k_2 equal to zero implies that the non-compressed concrete is completely ignored whereas assuming k_2 equal to unity implies that the torsional strength of non-compressed concrete remains undiminished in spite of the presence of flexural cracks. Hence values of k_2 equal to zero and unity may be expected to provide the lower bound and the upper bound respectively of the torsional strength. This is discussed further in Section 7-2 of the following chapter.

TABLE 6-7

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN COMBINED BENDING AND TORSION IN THIS INVESTIGATION

Beam	Actual Size		Concrete Strength		Longitudinal Bars**		Ties**		b' in	d' in	d in	M_u	T_{c1}	T_{c2} $k_2=1$	T_s	Tu					**** Type of Failure
																Analytical				Test	
	b_o in	d_o in	f'_c psi	τ_o* psi	Compr-ession	Ten-sion	Size	Spacing in								$k_2=0$	$k_2=1/2$	$k_2=3/4$	$k_2=1$		
												inch - kips									
A-2	6.50	12.20	4790	285	Nil	Nil	Nil	-	-	--	--	42	--	--	--	60***				53	Cl
A-3	6.10	12.20	4500	284	Nil	Nil	Nil	-	-	--	--	35	--	--	--	54***				53	Cl
A-4	6.05	12.20	5200	291	Nil	Nil	Nil	-	-	--	--	25	--	--	--	54***				52	Cl
B-2	6.20	12.20	4640	265	2-#3	2-#4	#3	6	4.6	10.6	11.0	195	23	41	13	36	57	67	77	72	Cl
B-3	6.05	12.20	4690	263	2-#3	2-#4	#3	6	4.6	10.6	11.0	110	20	39	34	54	74	83	93	95	Cl
C-1	6.00	12.20	4980	281	2-#3	2-#5	#3	4.5	4.6	10.6	10.9	280	27	41	15	42	63	73	83	78	Cl
C-2	6.15	12.20	4820	275	2-#3	2-#5	#3	4.5	4.6	10.6	10.9	195	25	42	40	65	86	97	107	105	Cl
C-3	6.10	12.20	5340	307	2-#3	2-#5	#3	4.5	4.6	10.6	10.9	110	23	46	69	92	115	127	138	111	Cl
D-1	6.20	12.20	4880	283	2-#5	4-#6	#3	4.5	4.6	10.6	10.1	637	30	44	37	67	89	100	111	99	Hy
D-2	6.20	12.20	5100	283	2-#5	4-#6	#3	4.5	4.6	10.6	10.1	365	37	44	95	132	154	165	176	164	Cl
D-3	6.20	12.20	4650	277	2-#5	4-#6	#3	4.5	4.6	10.6	10.1	195	30	43	95	125	147	157	168	156	Cl
E-1	6.20	12.20	4910	260	2-#4	2-#4	#3	4.5	4.6	10.6	11.0	195	23	41	19	42	63	73	83	76	Cl
E-2	6.20	12.20	5100	282	2-#4	2-#4	#3	4.5	4.6	10.6	11.0	110	21	44	48	69	91	102	113	101	Cl

* From TABLE 6-1

** Strength properties of the reinforcing steel are given in TABLE A-5, Appendix A

*** Computed from Equation (5-1)

**** Cl = Cleavage (tension) type failure

Hy = Hybrid: Shear (compression) type for the compressed concrete and cleavage (tension) for the non-compressed concrete

TABLE 6-8

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN COMBINED BENDING AND TORSION BY NYLANDER (1945)

Beam	Actual Size		Concrete Strength		Longitudinal Bars		Yield Stress ksi	Ties	d in	M_u	T_{c1}	T_{c2} $k_2=1$	T_s	Tu				*** Type of Failure
	b_o in	d_o in	f'_c* psi	τ_o** psi	Compression	Tension								Analytical $k_2=0$	Analytical $k_2=1/2$	Analytical $k_2=1$	Test	
										inch - kips								
Series III																		
2a	3.54	7.88	3360	220	Nil	3-0.55 in	38.0	Nil	7.33	9	3	7	0	3	7	10	13	Cl
2b	3.54	7.88	3360	220	Nil	3-0.55 in	38.0	Nil	7.33	9	3	7	0	3	7	10	13	Cl
3a	3.54	7.88	3220	216	Nil	3-0.55 in	38.0	Nil	7.33	48	6	7	0	6	10	13	14	Cl
3b	3.54	7.88	3220	216	Nil	3-0.55 in	38.0	Nil	7.33	48	6	7	0	6	10	13	14	Cl
4a	3.54	7.88	3080	211	Nil	3-0.55 in	38.0	Nil	7.33	73	6	7	0	6	10	13	16	Hy
4b	3.54	7.88	3080	211	Nil	3-0.55 in	38.0	Nil	7.33	73	6	7	0	6	10	13	16	Hy
Series VIII																		
1	7.88	7.88	3730	232	Nil	2-0.392 in	43.3	Nil	7.29	52	11	33	0	11	28	44	39	Cl
2	7.88	7.88	3730	232	Nil	2-0.392 in	43.3	Nil	7.29	52	11	33	0	11	28	44	31	Cl
3	7.88	7.88	3730	232	Nil	2-0.392 in	43.7	Nil	7.29	58	11	33	0	11	28	44	39	Cl
4	7.88	7.88	3730	232	Nil	2-0.392 in	43.7	Nil	7.29	58	11	33	0	11	28	44	35	Cl
5	7.88	7.88	3550	226	Nil	2-0.392 in	49.7	Nil	7.29	76	12	32	0	12	28	44	55	Cl
6	7.88	7.88	3550	226	Nil	2-0.392 in	48.7	Nil	7.29	76	12	32	0	12	28	44	51	Cl
7	7.88	7.88	3900	237	Nil	2-0.392 in	47.0	Nil	7.29	110	14	34	0	14	31	48	51	Cl
8	7.88	7.88	3900	237	Nil	2-0.392 in	46.2	Nil	7.29	110	14	34	0	14	31	48	55	Cl
9	7.88	7.88	3770	233	Nil	2-0.392 in	38.7	Nil	7.29	58	11	34	0	11	28	45	31	Cl
10	7.88	7.88	3770	233	Nil	2-0.392 in	37.0	Nil	7.29	58	11	34	0	11	28	45	20	Cl

* Taken 95% of the compressive strength of 6-in cubes

** Computed from Equation (6-10)

*** Cl = Cleavage (tension) type failure

Hy = Hybrid: Shear (compression) type for the compressed concrete and cleavage (tension) type for non-compressed concrete

TABLE 6-9

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN COMBINED BENDING AND TORSION BY COWAN AND ARMSTRONG (1955)

Beam	Actual Size		Concrete Strength		Longitudinal Bars***		Ties***		b'	d'	M_u	T_{c1}	T_{c2}	T_s	Tu				**** Type of Failure
															Analytical			Test	
	b_o in	d_o in	f_c'* psi	τ_o** psi	Compr-ession	Tension	Size	Spacing in	in	in			$k_2=1$		$k_2=0$	$k_2=1/2$	$k_2=1$		
											inch - kips								
R1	6	9	8120	342	2-#4	3-#4	#2	4	4.5	7.5	258	23	35	6	29	47	64	43	Cl
R2	6	9	8170	344	2-#4	3-#4	#2	4	4.5	7.5	158	20	36	11	31	49	67	79	Cl
R5	6	9	9500	371	2-#4	3-#4	#2	4	4.5	7.5	75	17	38	11	28	47	66	75	Cl
S1	6	9	8040	341	2-#4	3-#4	#2	4	4.5	7.5	207	21	35	6	27	45	62	83	Cl
S2	6	9	7770	335	2-#4	3-#4	#2	4	4.5	7.5	258	23	35	6	29	47	64	65	Cl

* Taken 95% of the cube strength
** Computed from Equation (6-10)
*** Yield stress: #4 bars 48.5 ksi and #2 bars 20.8 ksi
**** Cl = cleavage (tension) type failure

TABLE 6-10

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN COMBINED BENDING AND TORSION BY CHINENKOV (1959)

Beam	Actual Size		Concrete Strength		Yield Stress ksi***		b'	d'	d	M_u	T_{c1}	T_{c2}	T_s	Tu				**** Type of Failure
														Analytical			Test	
	b_o	d_o	f'_c*	τ_o**	Longitudinal Bars	Ties	in	in	in			$k_2=1$		$k_2=0$	$k_2=1/2$	$k_2=1$		
	in	in	psi	psi						inch - kips								
0.1	7.9	11.8	1450	145	54.2	41.0	6.2	10.1	10.4	486	0	27	23	23	37	50	49	Hy
0.1a	7.9	12.0	1450	145	52.7	41.0	6.2	10.1	10.6	469	0	28	28	28	42	56	47	Hy
0.2	7.9	12.2	1220	133	52.5	41.0	6.2	10.1	10.8	417	0	24	64	64	76	88	83	Hy
0.2a	7.9	12.0	1450	145	53.5	41.0	6.2	10.1	10.4	417	0	29	53	53	68	82	83	Hy
0.4B	7.9	12.0	1450	145	48.8	41.0	6.2	10.1	10.4	347	0	32	61	61	77	93	98	Hy
0.4	7.9	12.0	3310	219	52.5	41.0	6.2	10.1	10.4	365	37	52	70	107	133	159	146	Cl
0.4a	7.9	12.0	3310	219	52.5	41.0	6.2	10.1	10.4	347	36	52	70	106	132	158	139	Cl
0.4b	7.9	12.0	5130	273	52.5	41.0	6.2	10.1	10.6	365	41	65	70	111	144	176	146	Cl
0.4c	7.9	11.8	5130	273	55.0	41.0	6.2	10.1	10.4	382	41	64	70	111	143	175	153	Cl
0.4d	7.9	11.8	2700	197	52.7	41.0	6.2	10.1	10.4	313	31	46	70	101	124	147	125	Hy
0.4e	7.9	12.0	2700	197	52.7	41.0	6.2	10.1	10.6	330	32	47	70	102	126	149	132	Hy
0.4f	7.9	11.8	2700	197	53.0	35.8	6.3	10.2	10.4	347	30	46	109	139	162	185	139	Hy
0.4g	7.9	11.8	2700	197	51.2	35.8	6.3	10.2	10.4	435	26	46	53	79	102	125	98	Hy

* Taken 95% of the cube strength

** Computed from Equation (6-10)

*** Reinforcing steel: Longitudinal Bars 4 - 0.79 in. diameter, one in each corner
Ties 0.276 in. at 3.16 in. centers for beams 0.1 through 0.4e
and 0.394 in. at 3.16 in. centers for beams 0.4f and 0.4g

**** Cl = Cleavage (tension) type failure
Hy - Hybrid: Shear (compression) type for compressed concrete and cleavage (tension) type for the non-compressed concrete

TABLE 6-11

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN COMBINED BENDING AND TORSION BY GESUND AND BOSTON (1964)

Beam	Actual Size		Concrete Strength		Longitudinal Bars**		Ties	d	M_u	T_{c1}	T_{c2}	T_s	Tu				*** Type of Failure
													Analytical			Test	
	b_o in	d_o in	f'_c psi	τ_o psi	Compr-ession	Tension		in			$k_2=1$		$k_2=0$	$k_2=1/2$	$k_2=1$		
									inch - kips								
3	8	8	4360	251	2-#3	3-#3	Nil	7.0	58	13	38	0	13	32	51	58	Cl
4	8	8	4360	251	2-#4	3-#4	Nil	7.0	64	15	38	0	15	34	53	64	Cl
5	8	8	2320	183	2-#4	3-#4	Nil	7.0	86	13	27	0	13	27	40	43	Cl
6	8	8	2320	183	2-#4	3-#4	Nil	7.0	108	14	27	0	14	28	41	36	Cl
7	8	8	5590	284	2-#4	3-#4	Nil	7.0	177	21	42	0	21	42	63	59	Cl
8	8	8	5590	284	2-#4	3-#4	Nil	7.0	195	21	42	0	21	42	63	49	Cl
9	6	12	4700	261	2-#4	3-#4*	Nil	10.0	83	19	37	0	19	38	56	42	Cl
10	6	12	2800	201	2-#4	3-#4	Nil	10.0	156	19	29	0	19	34	48	39	Cl

* Beam 9, in addition, had two #4 longitudinal bars at middepth, one near each side
** Yield stress: #4 bars 51.0 ksi and #3 bars 50.0 ksi
*** Cl = Cleavage (tension) type failure

TABLE 6-12

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN COMBINED BENDING AND TORSION BY GESUND, SCHUETTE, BUCHANAN AND GRAY (1964)

Beam	Actual Size		Concrete Strength		Longitudinal Bars*		Ties*		b'	d'	d	M_u	T_{c1}	T_{c2}	T_s	Tu				**
																Analytical			Test	Type of Failure
	b_o in	d_o in	f'_c psi	τ_o psi	Compr-ession	Ten-sion	Size	Spacing in	in	in	in	inch - kips		$k_2=1$		$k_2=0$	$k_2=1/2$	$k_2=1$		
1	8	8	5030	270	2-#4	3-#4	#3	5	5.9	5.9	6.5	79	17	40	7	24	44	64	79	C1
2	8	8	5300	277	2-#4	3-#4	#3	2	5.9	5.9	6.5	102	18	41	63	81	102	122	102	C1
3	8	8	5310	277	2-#4	3-#4	#3	5	5.9	5.9	6.5	122	19	41	7	26	47	67	61	C1
4	8	8	4680	260	2-#4	3-#4	#3	2	5.9	5.9	6.5	134	19	39	40	59	79	98	67	C1
5	8	8	4240	248	2-#4	3-#4	#3	5	5.9	5.9	6.5	147	18	37	7	25	44	62	49	C1
6	8	8	4060	242	2-#4	3-#4	#3	2	5.9	5.9	6.5	168	19	36	21	40	58	76	56	C1
7	8	8	5280	276	2-#4	3-#4	#3	5	5.9	5.9	6.5	173	20	41	7	27	48	68	43	C1
8	8	8	5740	288	2-#4	3-#4	#3	2	5.9	5.9	6.5	176	21	43	17	38	60	81	44	C1
9	6	12	4860	265	2-#4	3-#4	#3	8	3.4	8.9	10.0	120	21	38	6	27	46	65	60	C1
10	6	12	3900	238	2-#4	3-#4	#3	8	3.4	8.9	10.0	176	22	34	6	28	45	62	44	C1
11	6	12	4860	265	2-#4	3-#4	#3	4	3.4	8.9	10.0	138	22	38	30	52	71	90	68	C1
12	6	12	3900	237	2-#4	3-#4	#3	4	3.4	8.9	10.0	213	23	34	23	46	63	80	53	C1

* Yield stress: #4 bars 51.0 ksi and #3 bars 50.0 ksi

** Cl = Cleavage (tension) type failure

6-5 Correlation of Tests in Combined Bending, Torsion and Shear

Twelve beams were tested by the author in combined bending, torsion and shear. Three were of plain concrete and nine were reinforced by longitudinal bars and transverse ties. The analytical and test results for these beams are presented in TABLE 6-13.

The torsional strength of plain concrete beams was calculated using Equation (5-25) and ignoring the presence of flexure.

The torsional strength of reinforced concrete beams was calculated using the analysis for combined bending and torsion to see whether transverse shear had any effect on the torsional strength. The average value of the bending moment over the gage length was used in the computations. The ultimate shear force and the nominal ultimate shear stress computed from Equation (6-13) are given in TABLE 6-13.

$$v_u = \frac{V_u}{b_o d} \tag{6-13}$$

where v_u = nominal ultimate shear stress due to transverse shear in combined loading

V_u = ultimate shear force in combined loading

6-6 Interaction Diagrams

The interaction diagrams for the beams tested in this investigation are shown in FIGURES 6-1 through 6-8. The interaction diagram for Beams A-1 through A-5 shown in FIGURE 6-1(a) and the corresponding non-dimensional plot of FIGURE 6-1(b) suggest that flexure has no signicant effect on the torsional strength of plain concrete beams. This is in conformity with the analysis presented in the preceding chapter. The inter-

TABLE 6-13

COMPARISON OF ANALYTICAL AND TEST RESULTS:

TESTS IN COMBINED BENDING, TORSION AND SHEAR IN THIS INVESTIGATION

Beam	Actual Size		Concrete Strength		Longitudinal Bars**		Ties**		b'	d'	d	*** M_u	T_{c1}	T_{c2}	T_s	Tu					V_u	v_u	***** Type of Failure
	b_o in	d_o in	f'_c psi	T_o* psi	Compression	Tensile	Size	Spacing in	in	in	in			$k_2=1$		Analytical $k_2=0$	Analytical $k_2=1/2$	Analytical $k_2=3/4$	Analytical $k_2=1$	Test	kips	psi	
												inch - kips											
A-6	6.20	12.20	5200	291	Nil	Nil	Nil	-	--	--	--	33	--	--	--	53****				52	1.6	21	Cl
A-7	6.10	12.20	4850	272	Nil	Nil	Nil	-	--	--	--	41	--	--	--	46****				40	2.0	27	Cl
A-8	6.20	12.20	3740	239	Nil	Nil	Nil	-	--	--	--	24	--	--	--	44****				43	1.1	15	Cl
F-1	6.10	12.20	5060	263	3-#3	3-#3	#3	8.0	4.63	10.63	11.0	140	21	40	16	37	57	67	77	58	9.2	137	Cl
F-2	6.10	12.20	5060	263	3-#3	3-#3	#3	8.0	4.63	10.63	11.0	95	19	40	33	52	72	82	92	83	6.2	92	Cl
F-3	6.15	12.20	4650	253	3-#3	3-#3	#3	8.0	4.63	10.63	11.0	50	16	39	33	49	69	78	88	89	3.2	47	Cl
G-1	6.20	12.20	4560	247	3-#3	3-#3	#2	3.5	4.75	10.75	11.2	155	21	39	21	42	62	71	81	73	10.2	147	Cl
G-2	6.20	12.20	4560	247	3-#3	3-#3	#2	3.5	4.75	10.75	11.2	95	18	39	48	66	86	95	105	92	6.2	89	Cl
G-3	6.15	12.20	4920	249	3-#3	3-#3	#2	3.5	4.75	10.75	11.2	50	16	38	64	80	99	109	118	103	3.2	47	Cl
H-1	6.10	12.20	4450	244	Nil	2-#3& 1-#6	#2	3.5	4.75	10.75	11.0	245	25	37	12	37	56	65	74	74	16.2	242	Cl
H-2	6.10	12.20	4450	244	Nil	2-#3& 1-#6	#2	3.5	4.75	10.75	11.0	155	22	37	12	34	53	62	71	84	10.2	152	Cl
H-3	6.25	12.20	4830	249	Nil	2-#3& 1-#6	#2	3.5	4.75	10.75	11.0	80	19	39	12	31	51	60	70	88	5.2	76	Cl

* From TABLE 6-1

** Strength properties of reinforcing steel are given in TABLE A-5, Appendix A

*** Average over the gage length

**** Computed from Equation (5-25)

***** Cl - Cleavage (tension) type failure

action diagrams for the reinforced concrete beams are shown in FIGURES 6-2 through 6-8. The factors for the non-dimensional interaction diagrams were computed both from the analytical and the test results. These factors are given in TABLE 6-14.

The interaction diagrams for beams in Groups B through E tested in combined bending and torsion are given in FIGURES 6-2 through 6-5. Two sets of non-dimensional factors were computed: (i) Ratio of ultimate torque in combined loading to the torsional strength of the equivalent plain concrete section in pure torsion, Tu/T_{co} , and (ii) Ratio of ultimate torque in combined loading to the ultimate torque of the section in pure torsion, Tu/Tuo. Since beams in the same group had identical reinforcement, the values of the ratio Tu/Tuo were obtained by dividing the ultimate torque of a beam subjected to combined loading by the ultimate torque of the last beam in the same group which was tested in pure torsion. This involved some approximation due to the variations in b_o , f_c' and τ_o . The ratios Tu/Tco and Tu/Tuo were calculated both from the analytical and the test results. The analytical and the test results were distinguished by superscripts a and t respectively. In general, the interaction diagrams based on test results and those based on the analysis follow similar trends. The agreement is strikingly close for beams in Groups D and E as shown in FIGURES 6-2, 6-3 and 6-5.

The interaction between bending and torsion, based on the analytical and test results for the beams in Groups F through H tested in combined bending, torsion and shear, is illustrated in FIGURES 6-6 through 6-8. The lack of a close agreement between the analytical and test results for the beams in Group H is due to the underestimate of the term T_{s1} for these beams. In reality, the flexural compression tends to increase the

TABLE 6-14

FACTORS FOR NON-DIMENSIONAL INTERACTION DIAGRAMS

Beam	Mu (in. kips)	Muo (in. kips)	$\frac{M_u}{Muo}$	T_u^a $k_2=3/4$ (in. kips)	T_u^t (in. kips)	$\frac{T_u^t}{T_u^a}$	T_{co} in. kips	$\frac{T_u^a}{T_{co}}$	$\frac{T_u^t}{T_{co}}$	$\frac{T_u^a}{T_{uo}^a}$	$\frac{T_u^t}{T_{uo}^t}$
A-2	42	69	0.61	60	53	0.88	60	1.00	0.88	1.09	0.91
A-3	35	64	0.55	54	53	0.98	54	1.00	0.98	0.98	0.91
A-4	25	65	0.38	54	52	0.96	54	1.00	0.96	0.98	0.90
A-5	0	66	0	55	58	1.05	55	1.00	1.05	1.00	1.00
B-2	195	228	0.86	67	72	1.07	52	1.29	1.38	0.75	0.85
B-3	110	228	0.48	83	95	1.14	49	1.69	1.94	0.93	1.12
B-4	0	228	0	89	85	0.96	55	1.62	1.55	1.00	1.00
C-1	280	297	0.94	73	78	1.07	51	1.43	1.53	0.54	0.70
C-2	195	297	0.66	97	105	1.08	53	1.83	1.98	0.72	0.94
C-3	110	297	0.37	127	111	0.87	58	2.19	1.91	0.94	1.00
C-4	0	297	0	135	111	0.82	66	2.05	1.68	1.00	1.00
D-1	637	823	0.77	100	99	0.99	55	1.82	1.80	0.68	0.68
D-2	365	823	0.44	165	164	0.99	55	3.00	2.98	1.11	1.12
D-3	195	823	0.24	157	156	0.99	54	2.91	2.90	1.06	1.07
D-4	0	823	0	148	146	0.99	53	2.79	2.76	1.00	1.00
E-1	195	228	0.86	73	76	1.04	51	1.43	1.49	0.60	0.63
E-2	110	228	0.48	102	101	0.99	55	1.85	1.84	0.84	0.83
E-3	0	228	0	122	121	0.99	53	2.30	2.28	1.00	1.00
A-6	33	67	0.49	53	52	0.98	57	0.93	0.91	1.26	1.24
A-7	41	62	0.66	46	40	0.87	51	0.91	0.78	1.10	0.95
A-8	24	55	0.44	44	43	0.98	47	0.94	0.91	1.05	1.02
A-9	0	50	0	42	42	1.00	42	1.00	1.00	1.00	1.00
F-1	140	187	0.75	67	58	0.87	50	1.34	1.16	0.83	0.61
F-2	95	187	0.51	82	83	1.01	50	1.64	1.66	1.01	0.87
F-3	50	187	0.27	78	89	1.14	49	1.59	1.82	0.96	0.94
F-4	0	187	0	81	95	1.17	48	1.69	1.98	1.00	1.00
G-1	155	190	0.82	71	73	1.03	48	1.48	1.52	0.63	0.63
G-2	95	190	0.50	95	92	0.97	48	1.98	1.92	0.85	0.80
G-3	50	190	0.26	109	103	0.94	48	2.27	2.15	0.97	0.90
G-4	0	190	0	112	115	1.03	48	2.33	2.40	1.00	1.00
H-1	245	348	0.70	65	74	1.14	46	1.41	1.61	1.06	1.16
H-2	155	348	0.45	62	84	1.35	46	1.35	1.83	1.02	1.31
H-3	80	348	0.23	60	88	1.47	49	1.22	1.80	0.98	1.38
H-4	0	348	0	61	64	1.05	49	1.24	1.31	1.00	1.00

Average	1.03
Mean Deviation	0.09

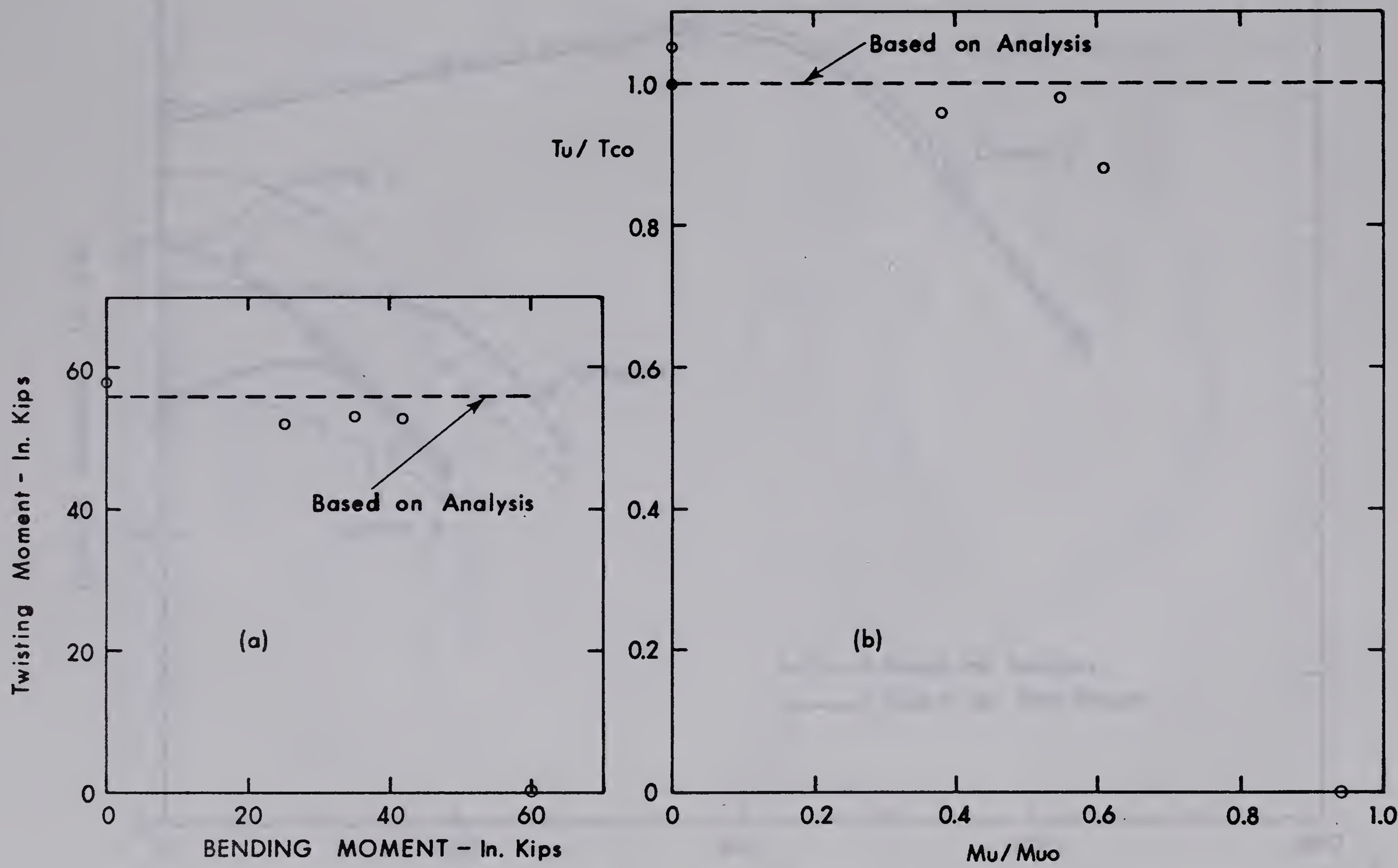


FIGURE 6-1 INTERACTION DIAGRAMS FOR BEAMS A-1 THROUGH A-5

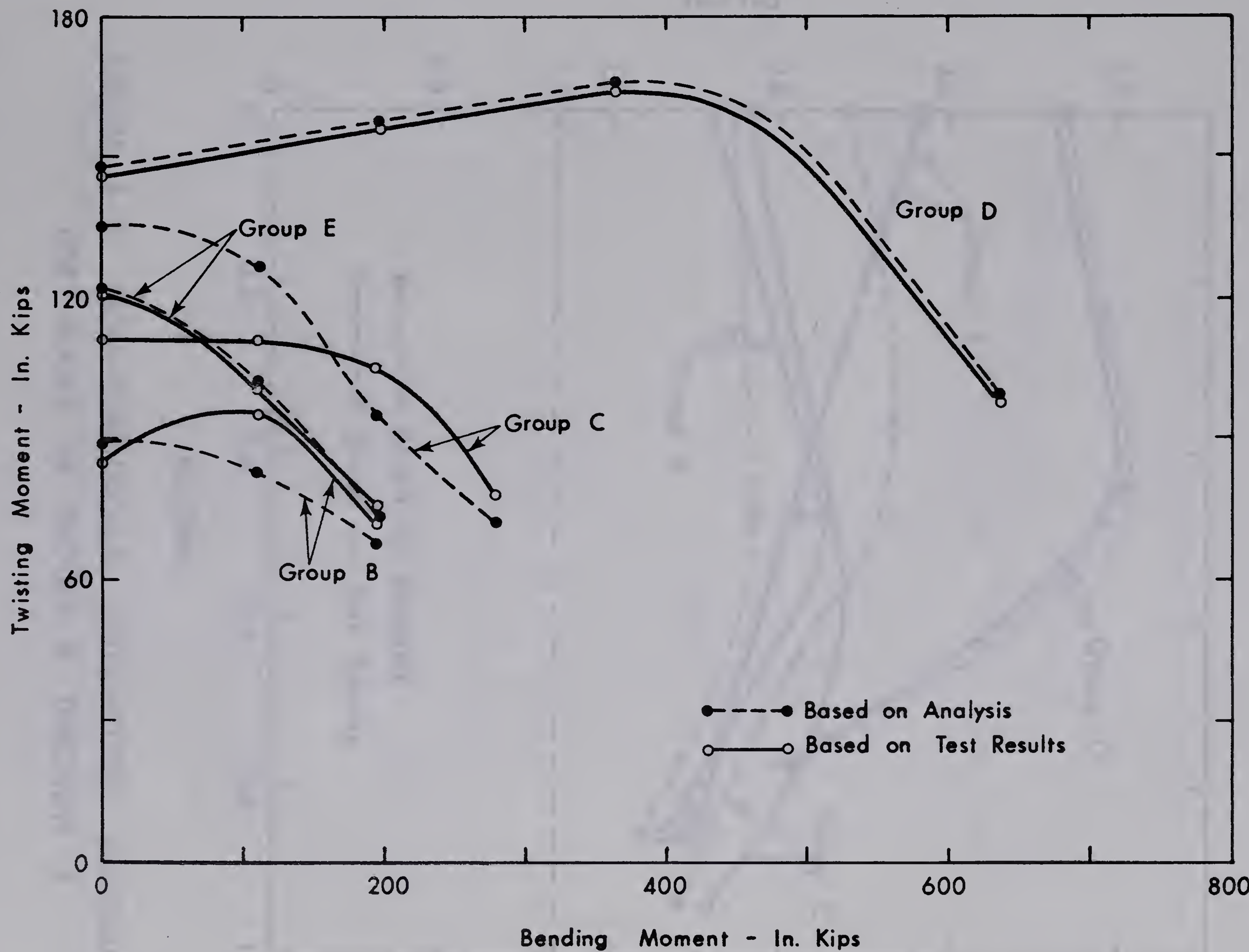
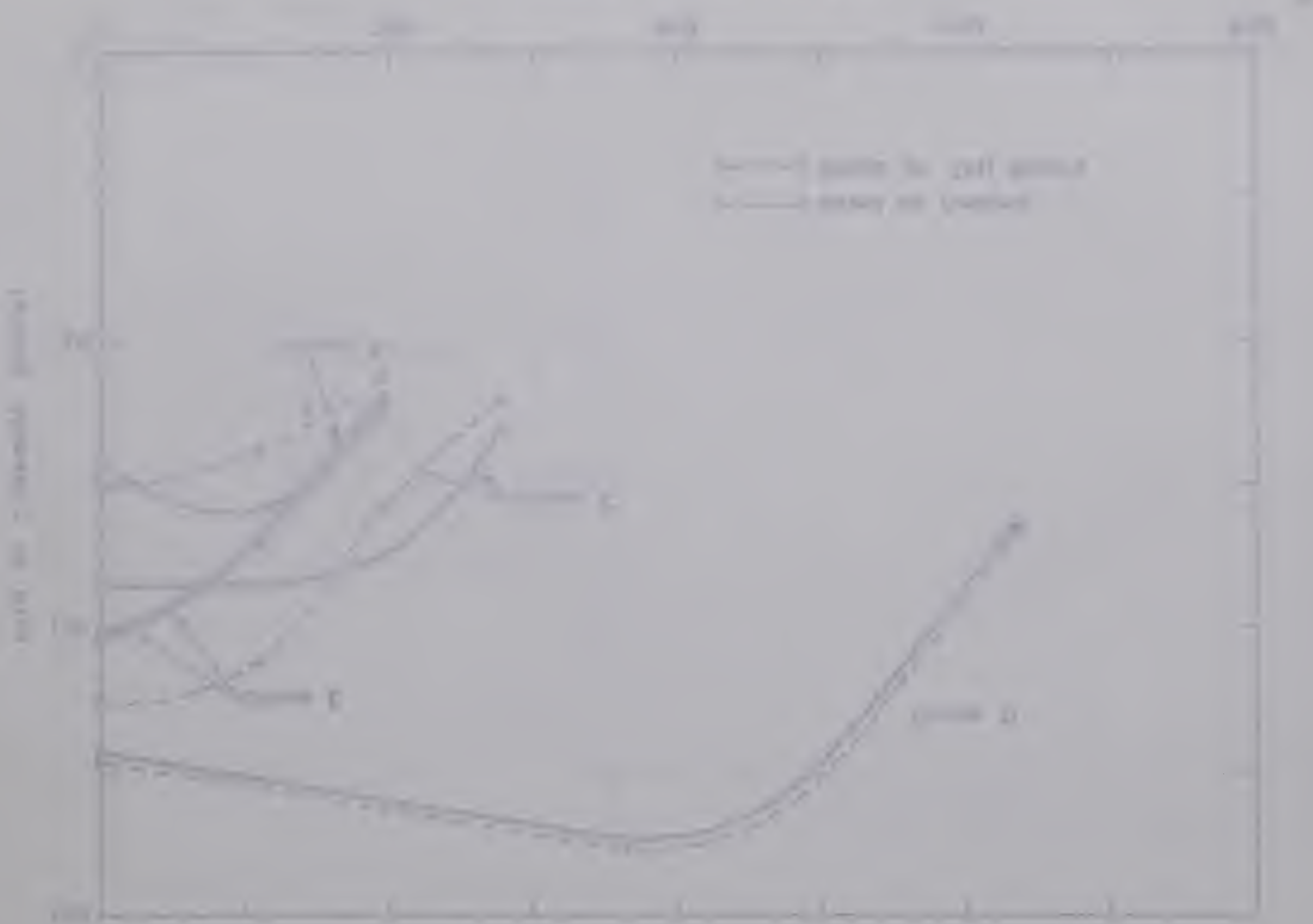


FIGURE 6-2 INTERACTION DIAGRAMS FOR BEAMS IN GROUPS B THROUGH E



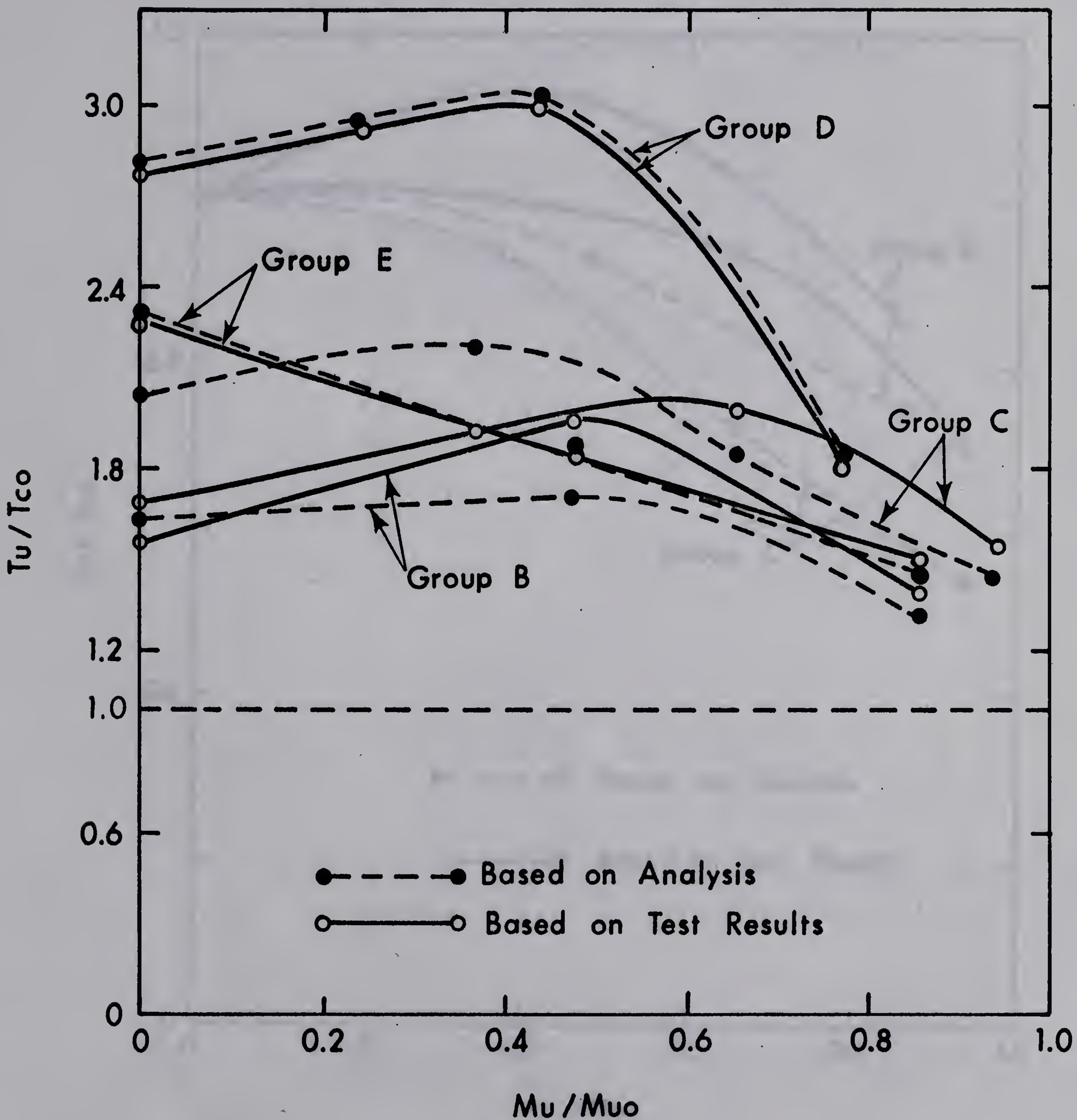


FIGURE 6-3 NON-DIMENSIONAL INTERACTION DIAGRAMS FOR BEAMS IN GROUPS B THROUGH E

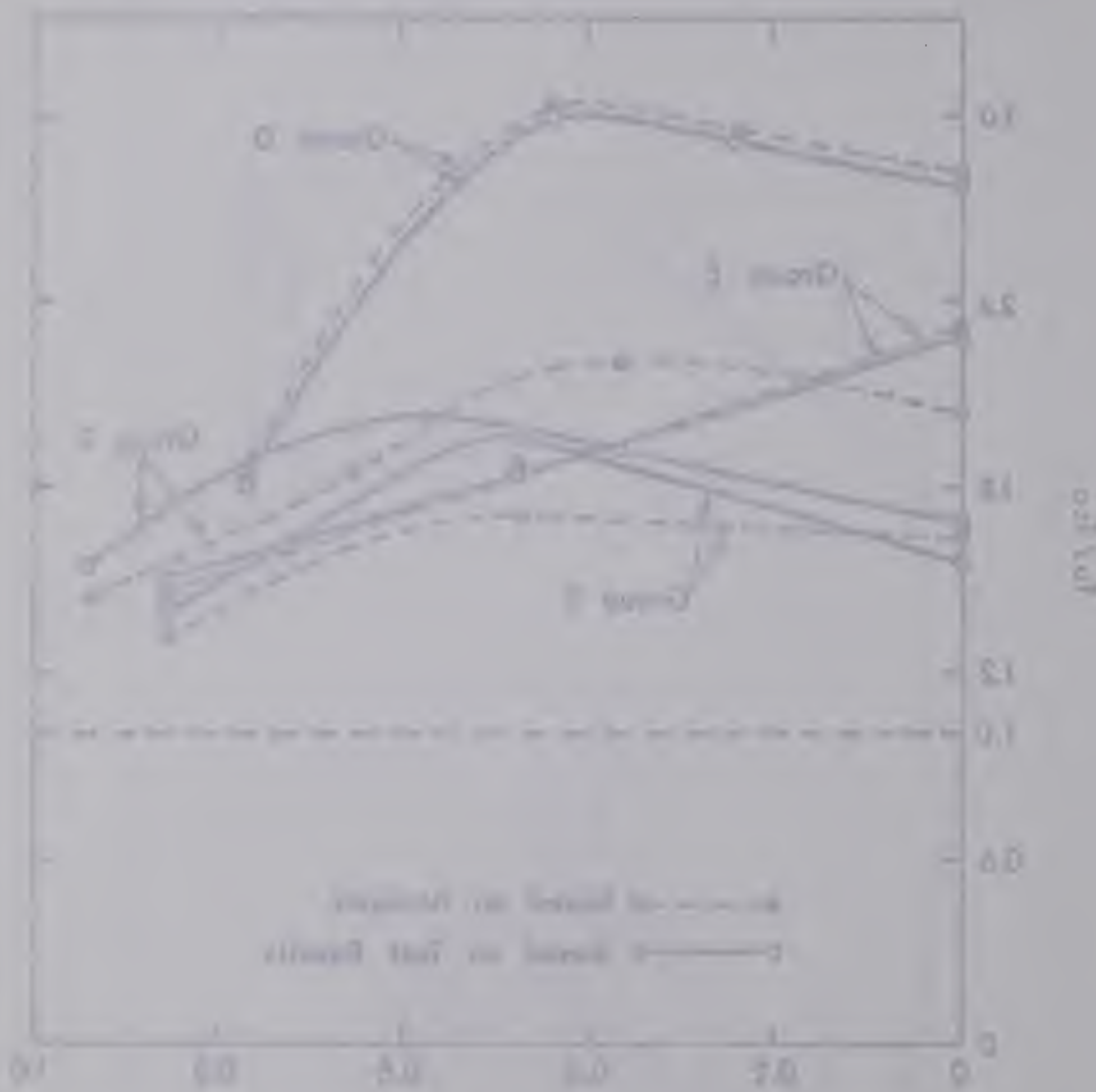


FIGURE A-2. NON-DIMENSIONAL INTERACTION DIAGRAMS FOR BEAMS IN GROUPS B THROUGH E

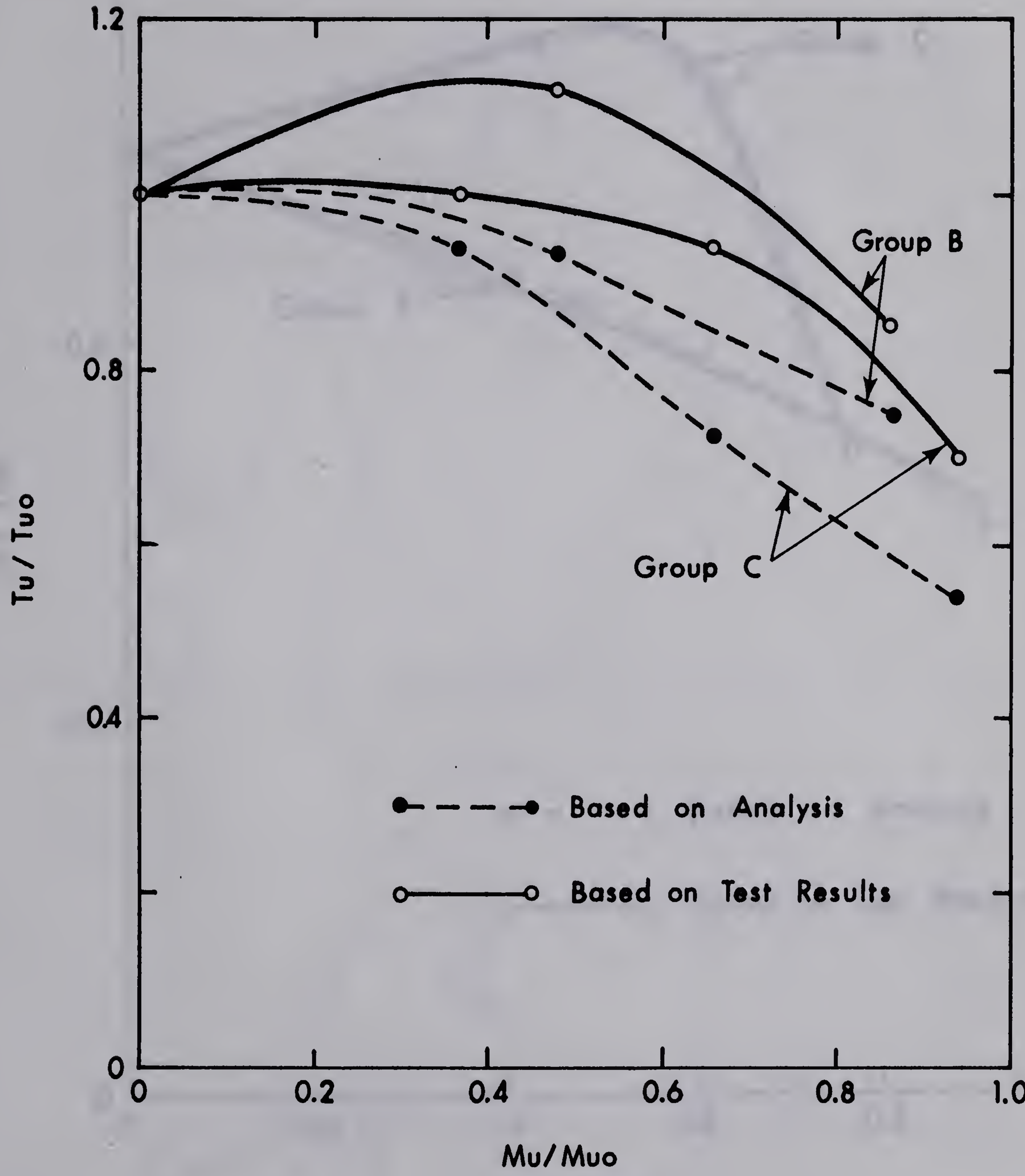


FIGURE 6-4 NON-DIMENSIONAL INTERACTION DIAGRAMS FOR BEAMS IN GROUPS B AND C

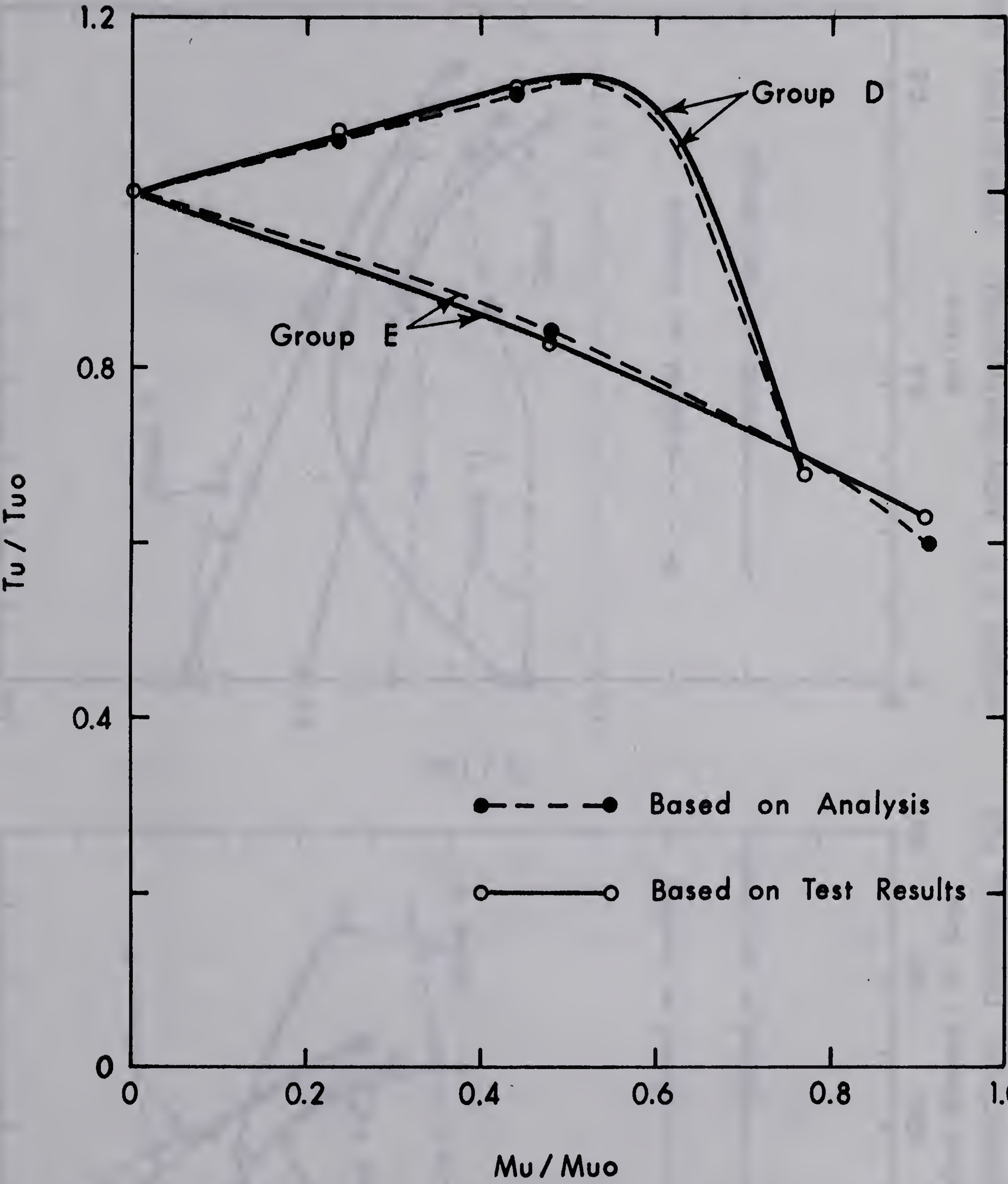


FIGURE 6-5 NON-DIMENSIONAL INTERACTION DIAGRAMS FOR BEAMS IN GROUPS D AND E

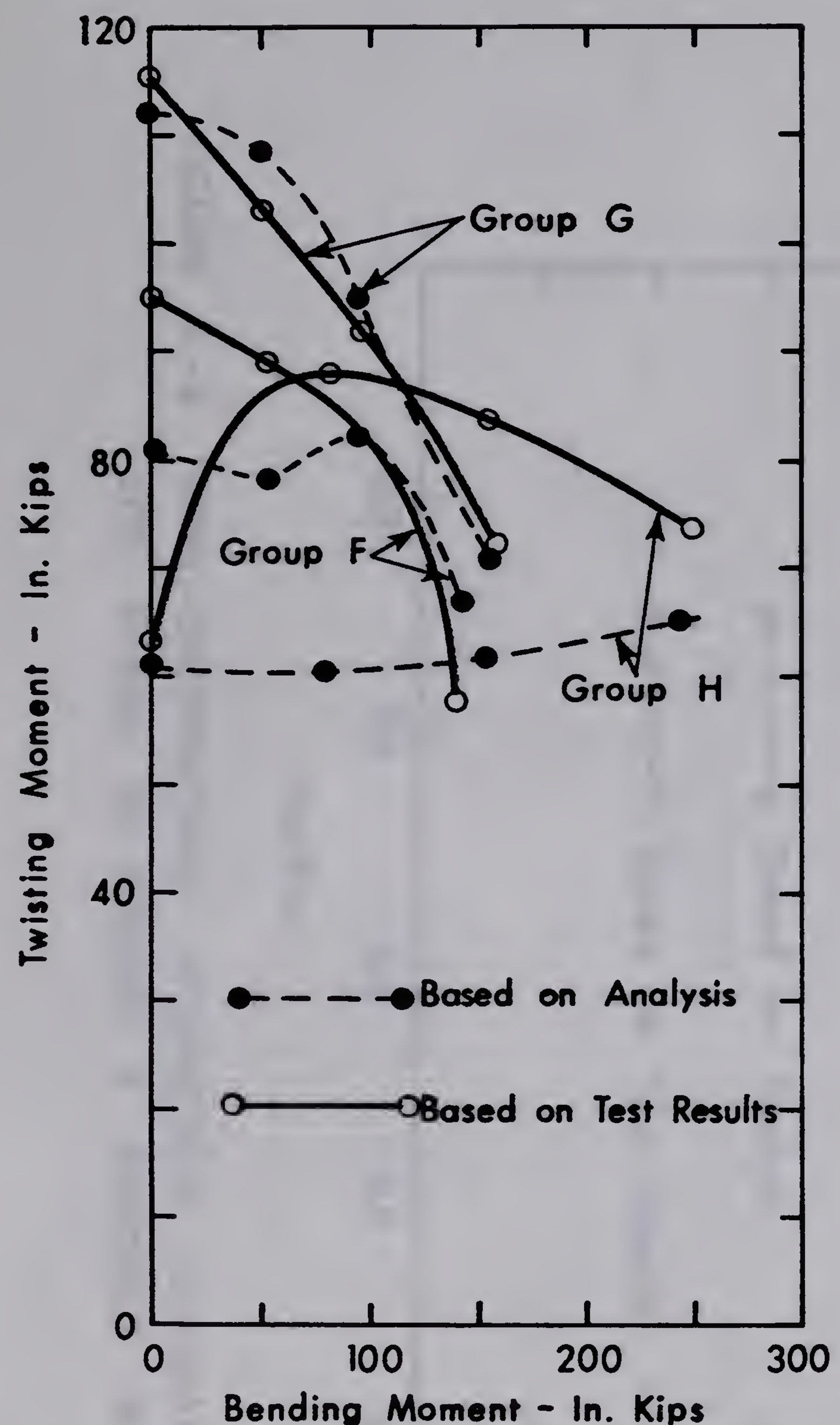


FIGURE 6-6 INTERACTION DIAGRAMS FOR BEAMS IN GROUPS F THROUGH H

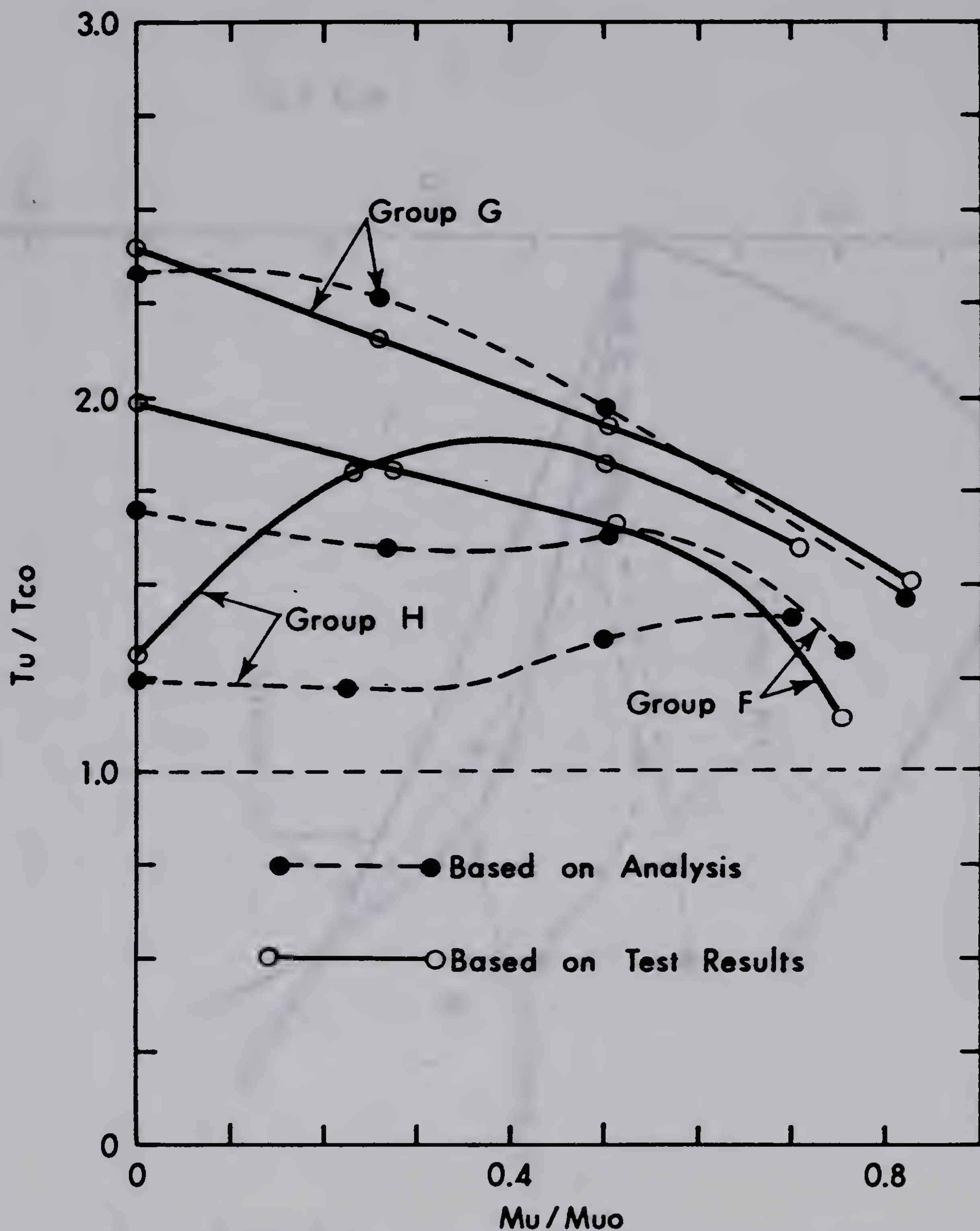


FIGURE 6-7 NON-DIMENSIONAL INTERACTION DIAGRAMS FOR BEAMS IN GROUPS F THROUGH H

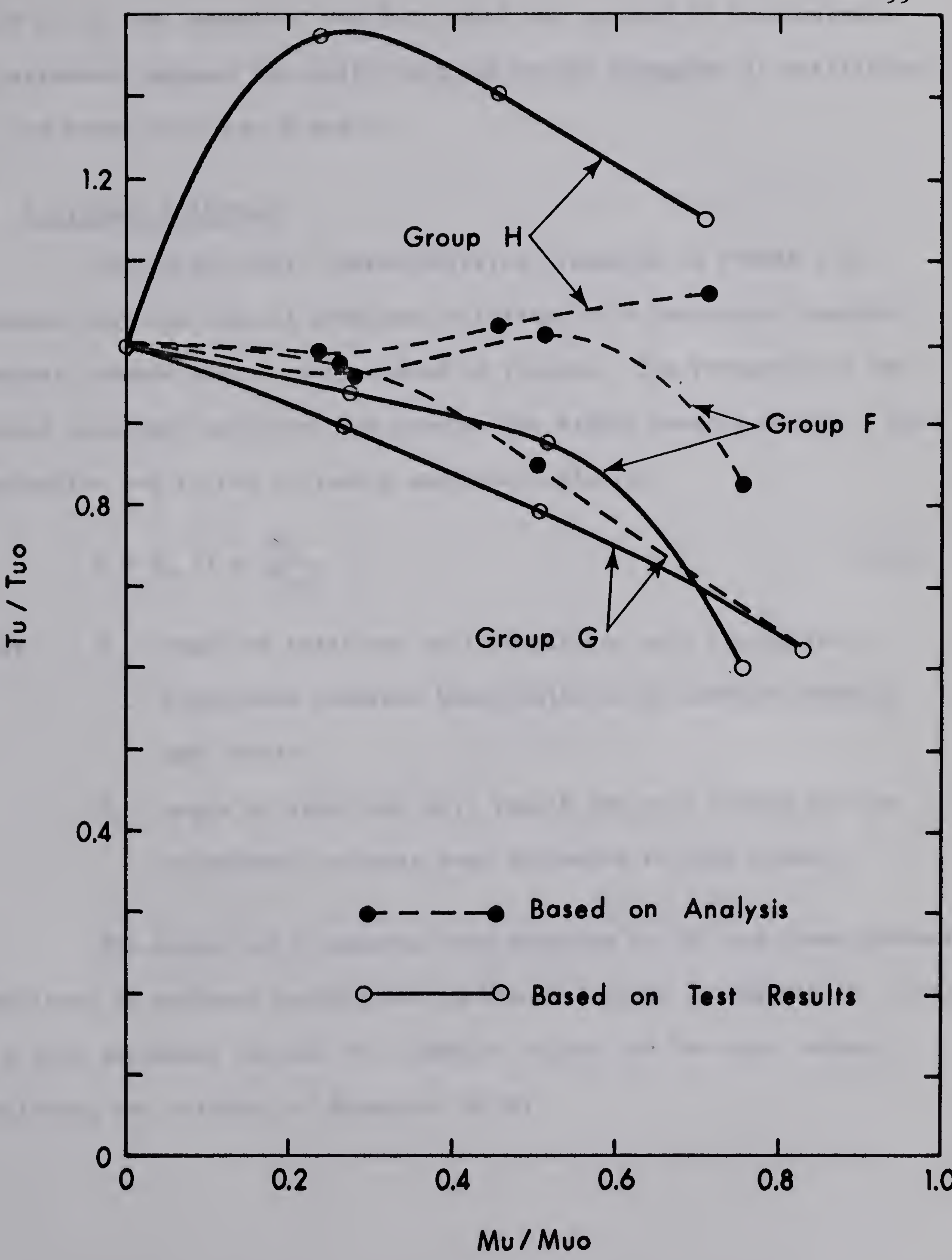


FIGURE 6-8 NON-DIMENSIONAL INTERACTION DIAGRAMS FOR BEAMS IN GROUPS F THROUGH H

value of T_{s1} for Beams H-2 and H-3, which was ignored in the analysis. The agreement between the analytical and actual strengths is satisfactory for the beams in Groups F and G.

6-7 Torsional Stiffness

The torque-twist characteristics presented in FIGURE 4-12 indicate that the initial torsional stiffness of a reinforced concrete beam was reduced due to the presence of flexure. The reduction in the initial torsional stiffness was greater for higher bending moment. This observation led to the following empirical relation.

$$\theta = \theta_o \left(1 + \frac{M_u}{M_{uo}}\right) \tag{6-14}$$

where θ = angle of twist per unit length per unit torque for a reinforced concrete beam subjected to combined bending and torsion

θ_o = angle of twist per unit length per unit torque for the reinforced concrete beam subjected to pure torsion

The values of θ computed from Equation (6-14) and those determined from tests in combined bending and torsion are given in TABLE 6-15. There is a fair agreement between the computed values and the test values justifying the validity of Equation (6-14).

TABLE 6-15

EFFECT OF FLEXURE ON TORSIONAL STIFFNESS

Beam	$\frac{Mu}{M_{uo}}$ *	θ** Computed***	θ** Test****	$\frac{\theta(\text{Test})}{\theta\ (\text{Computed})}$
B-2	0.86	2.68	2.41	0.90
B-3	0.48	2.13	1.74	0.82
B-4	0	1.44	1.44	1.00
C-1	0.94	2.15	2.32	1.08
C-2	0.66	1.84	1.74	0.95
C-3	0.37	1.52	1.42	0.93
C-4	0	1.11	1.11	1.00
D-1	0.77	2.32	3.63	1.56
D-2	0.44	1.89	1.22	0.65
D-3	0.24	1.62	1.62	1.00
D-4	0	1.31	1.31	1.00
E-1	0.86	2.44	2.42	0.99
E-2	0.48	1.94	1.62	0.84
E-3	0	1.31	1.31	1.00

Average value of $\frac{\theta\ (\text{test})}{\theta\ (\text{computed})}$ = 0.98

Mean Deviation = 0.14

* From TABLE 6-14.
** In radians per inch x 10^6 for a torque of one in. kip.
*** Computed from Equation (6-14) using the value of θ_o obtained from the pure torsion test.
**** Determined from the initial slope of the torque-twist curves.

CHAPTER VII

DISCUSSION

7-1 Behavior

The behavior of plain and reinforced concrete beams tested in combined loading in this investigation is reported in Section 4-2. Detailed information on the behavior of individual specimen is included in Appendix B.

Two distinct modes of failure were observed in this investigation.

(i) a cleavage (tension) type failure, and

(ii) a hybrid failure

A cleavage (tension) type failure is characterised by the development of diagonal cracks due to torsion on all faces of the specimen. It implies the overcoming of flexural compression by the diagonal tension set up due to torsion. In this type of failure, the whole section is in diagonal tension at the point of failure which occurs due to cleavage.

All beams tested in this investigation, except Beam D-1, failed due to cleavage. There was ample evidence of the overcoming of flexural compression in all beams which failed due to cleavage. The development of diagonal cracks due to torsion on the top face and the reduction in the flexural compressive stress in the top longitudinal steel often leading to stress reversal as the torque was increased were clear indications of this effect. There was almost complete absence of debris which is typical of cleavage type of failure.

The hybrid failure is characterised by the shear (compression) failure of the concrete in the zone of flexural compression and cleavage of the concrete not compressed due to flexure. The possibility of this type of failure exists when the flexural compression is extremely high. In the analysis of CHAPTER V, it is assumed that the concrete in flexural compression fails due to shear (compression) if the flexural compression exceeds the critical value of normal stress given by Equation (C-13), Appendix C. The hybrid failure is usually associated with the formation of debris and an increase in the compressive strain in the top longitudinal steel as the torque increases.

Only Beam D-1, which was subjected to the maximum flexural load, had this type of failure. The application of torsion to this beam did not produce diagonal cracks on the top face. Instead, it precipitated crushing and complete destruction of the zone in flexural compression. The flexural compressive strain in the top longitudinal steel continued to increase as the torque was increased.

Failure due to cleavage is by far the most common type of failure in combined loading. The modes of failure predicted by the analysis of CHAPTER V are indicated in TABLES 6-7 through 6-13. There was no evidence of the formation of plastic hinges on the top or the sides of the beam as suggested by Lessig (1959). In all beams, except Beam D-1, diagonal cracks developed on the top and on both vertical faces of the beam before failure.

7-2 Strength

An analysis of strength has been presented in CHAPTER V. In

this analysis the most general case is that of a beam with longitudinal bars and transverse ties subjected to combined bending and torsion. The following may then be considered as particular or limiting cases and may be derived from the general case.

(i) the case of pure torsion

(ii) the case of a reinforced concrete beam with longitudinal steel only.

A desirable feature of any analysis is that it be valid for the limiting cases. Also it is desirable that the transition from one case to another or from one mode of failure to another be gradual and defined. The analysis of CHAPTER V has these desirable features. It is of interest to indicate that some of the analyses presented by other investigators do not have these features.

Certain approximations and idealisations have been introduced in the strength analysis of CHAPTER V. The approximation involved in the modified sand heap analogy has already been indicated in Section 5-4. The action of the longitudinal bars in resisting torsion has been idealised. This is evident from the derivation of the term T_{s2}. The actual action of the longitudinal bars in resisting torsion is more complex. The secondary effects, like continuity, shear and axial force, have been ignored in the derivation of the expression for the lateral force developed by a longitudinal bar. Since T_{s2} forms only a small part of the total torsional strength, the idealisation of the action of longitudinal bars is justified.

The contribution of concrete in resisting torsion either in pure torsion or in combined loading has been quite controversial. Gesund and Boston (1964) completely disregarded the contribution of concrete whereas

Lessig (1959) disregarded the concrete in flexural tension. Chinenkov (1959) pointed out that the reason for the computed loads being smaller than the test loads in his investigation apparently lies in the resistance offered by the uncracked concrete in the interior of the section in resisting torsion. There is no doubt that the torque resisting capacity of the non-compressed concrete is reduced due to the presence of flexural cracks. Hence the assumption that the torsional capacity of non-compressed concrete remains undiminished in spite of the presence of flexural cracks may be expected to yield an unconservative estimate of the torsional strength. On the other hand, complete disregard of the non-compressed concrete may be expected to yield a conservative estimate of torsional strength. The participation of non-compressed concrete in resisting torsional stresses is indicated by the following observations.

(i) The magnitude of flexure had no significant effect on the value of the cracking torque. This can be seen from the torque-twist curves shown in FIGURE B-12, Appendix B. Since the value of cracking torque depends on the torque resisting capacity of concrete alone, the cracking torque would have been seriously reduced if the non-compressed concrete were excluded from participation in resisting torsional stresses.

(ii) Torsion produced diagonal cracks which crossed the vertical flexural cracks as shown in FIGURES 4-1 through 4-4. The fact that torsion produced these diagonal cracks in the portion of non-compressed concrete, which previously had flexural cracks, indicates that the non-compressed concrete participates in resisting torsional stresses.

From the above discussion, it appears that the non-compressed concrete participates in resisting torsional stresses though its strength

is somewhat reduced due to the presence of flexural cracks. Hence values of k_2 equal to zero and unity may be expected to give respectively the lower bound and the upper bound of the torsional strength. Tests in combined bending and torsion for all beams, except Beam B-3, confirm the above concept of upper and lower bounds of torsional strength as shown in TABLE 6-7. The tests by other investigators, analysed in TABLES 6-8 through 6-12, are also generally in agreement with the above concept. Tests by Chinenkov, analysed in TABLE 6-10, also provide a reasonable confirmation of the concept developed above.

The presence of flexure may increase or decrease the torsional strength depending upon the properties of the beam section and the level of bending moment. For example, the effect of flexure is different for the following two types of beam sections.

(i) a typical beam section with most of the longitudinal steel located in the zone of flexural tension.

(ii) a section with the longitudinal steel distributed equally near the top face and the bottom face of a beam.

According to the analysis developed in this investigation, the torsional strength in Case (i) is increased due to the presence of flexure up to a certain value which might be called the optimum value of flexural moment. The reason for this increase in the torsional strength is that while the value of T_{s1} is unaffected, the value of T_{c1} increases due to the presence of flexural compression.

In case (ii), however, the effect of flexure is generally to reduce the torsional strength unless an excessive amount of longitudinal steel is provided. In general, the reduction in T_{s1} due to the reduced

capacity of longitudinal steel in resisting the longitudinal component of diagonal tension is more than the corresponding increase in T_{c1}. The result is a reduction in the torsional strength of the section.

The two cases cited above are represented by the beams in Groups D and E respectively. For the beams in Group D, which correspond to Case (i), the longitudinal steel was mostly concentrated on the tension side whereas for beams in Group E, which correspond to Case (ii), the longitudinal steel was equally distributed on the tension side and the compression side. The interaction diagrams for the beams in these two groups are shown in FIGURE 6-5. This figure shows clearly how the shape of the interaction diagram may be influenced by the section properties of the beam. In addition to the striking agreement between the analytical and actual strengths, the analysis presented in this investigation reproduces the actual trends in the variation of torsional strength in combined bending and torsion for the two diverse cases of cross-sectional properties. This constitutes a fair confirmation of the concepts used in the development of the analysis of CHAPTER V.

The tests in combined bending, torsion and shear in this investigation were of an exploratory nature. The main objective of the tests was to study the effect of transverse shear on the torsional strength of beams subjected to combined loading. The tests, which are analysed in TABLE 6-13, suggest that the torsional strength is not affected significantly by transverse shear provided the beam has enough transverse steel to prevent shear failure in the absence of torsion. The agreement between analytical and actual strengths is reasonable for all beams analysed in TABLE 6-13 except for Beams H-2 and H-3. The reason for the difference

in analytical and actual strengths for Beams H-2 and H-3 has been discussed in Section 6-6.

It has been indicated above that values of k_2 equal to zero and unity give, respectively, the lower bound and the upper bound of the torsional strength. In the present tests, a value of k_2 equal to 3/4 was found to give a close agreement between analytical and actual strengths. The average value of the ratio of actual and analytical strengths is 1.03 and the mean deviation is 0.09 for all beams tested in combined loading in this investigation. This is shown in TABLE 6-14.

In general the agreement between analytical and actual strengths is better for beams with larger cross-sectional dimensions and for those beams which had higher amounts of reinforcing steel. The variation in strength due to the flaws in concrete is more critical for a beam having a smaller cross-section than that for a beam having a larger cross-section. For beams with a high proportion of reinforcing steel, the contribution of concrete towards torsional strength recedes in importance. Also the difference between the analytical and actual strengths is partly due to the error in estimating the true tensile strength of concrete. A better agreement may, therefore, be expected for beams with fairly high amounts of reinforcing steel.

7-3 Sequence of Loading

In this investigation, the transverse load was applied first and the specimens were then twisted to failure. Gardner (1960) used the same sequence of loading in his tests on prestressed concrete beams. Cowan and Armstrong (1955) and Gesund and Boston (1964) used testing procedures in which the bending and twisting moments increased in the

same proportion. This proportion was constant throughout any particular test though a new proportion could be chosen for another test. Chinenkov (1959) used a setup which permitted a change in the ratio of bending moment and twisting moment at any stage of a test. He varied this ratio during the test for some of his beams. Tests in which torsion precedes flexure have not been reported so far.

The question as to how the sequence of loading affects the strength and behavior of beams subjected to combined loading is of considerable importance. A comparison of the photographs of the crack patterns in this and other investigations shows that the mode of crack propogation and also the final pattern of cracks is influenced by the sequence of loading. In this investigation, torsion followed flexure. Hence, the final crack pattern consists of two sets of cracks. The first set comprises of vertical flexural cracks whereas the second set comprises of diagonal cracks due to torsion. The two sets are seen to cross each other at an angle of the order of 45° in FIGURES 4-1 through 4-4. The investigations, in which the torsional load and the flexural load were increased simultaneously, have indicated that the crack inclination depends on the ratio of torsion to flexure (Cowan and Armstrong, 1955).

Chinenkov (1959) assumed a constant crack angle on all sides of the specimen. He stated that although this assumption does not correspond to the actual pattern of crack propagation, it does not affect significantly the carrying capacity of the element. Tests analysed and presented in TABLES 6-7 through 6-12 involve different sequences of loading. Since there is a fair agreement between analytical and actual strengths irrespective of the sequence of loading, it appears that the sequence of loading does not affect the ultimate strength significantly. This is in a greement with Chinenkov's observations.

7-4 Influence of Variables

The analysis developed in CHAPTER V involves several variables which influence the computed value of strength of a beam. These variables may be grouped into the following four categories.

(i) Variables Relating to Concrete Strength

The tensile strength, f_t' , and the compressive strength, f_c' , influence the strength of a beam subjected to combined loading. In a cleavage (tension) type of failure, the torsional strength is governed essentially by the tensile strength, whereas, in a hybrid failure both the tensile strength and the compressive strength influence torsional strength. Since cleavage failure is more frequent in combined loading than hybrid failure, the tensile strength of concrete assumes greater importance than the compressive strength of concrete.

The contribution of concrete recedes in importance as the proportion of reinforcing steel is increased. Consequently, the influence of concrete strength diminishes for beams with high proportions of longitudinal steel and transverse steel. Since the contribution of concrete for such beams is only of the order of one-third of the torsional strength, the error in computed torsional strength associated with a 15 percent error in estimating the strength properties of concrete may be of the order of 5 percent. However, for beams with little or no steel, the error in the computed torsional strength may be of the same order as the error in estimating concrete strength.

(ii) Variables Relating to Steel Strength

The contribution of steel towards torsional strength depends on the yield stress of steel. The terms T_{s1} and T_{s2} are governed by the

yield stresses f_y , f_y' , and f_{yt} . The yield stresses f_y and f_y' do not influence the torsional strength if the proportion of longitudinal steel is high as compared to that of transverse steel. In such a case, the term T_{s1} varies linearly with the yield stress of transverse steel, f_{yt}. On the other hand, if the proportion of transverse steel is high as compared to that of longitudinal steel, the yield stress of transverse steel, f_{yt}, is inconsequential. In this case, the term T_{s1} is governed by f_y or f_y' depending upon whether the bottom longitudinal steel or the top longitudinal steel yields first.

The error in the measurement of yield stress of steel assumes greater importance for a beam with high proportions of longitudinal steel and transverse steel due to the increase in T_s. In such a case, a 6 percent error in the measurement of yield stress of steel may be associated with an error of the order of 4 percent in the computed torsional strength. The error in the computed torsional strength can, however, be greater if the flexural stresses in the longitudinal steel are of such magnitude that they inhibit the development of yield stress in the transverse tie because the required force to resist the longitudinal component of diagonal tension can not be mobilised. The contribution of steel to torsional strength and the influence of errors in the measurement of the yield stress of steel recede in importance for beams with low proportions of reinforcing steel.

(iii) <u>Variables Relating to Geometry of Cross-Section</u>

In addition to all the cross-sectional dimensions shown in FIGURE 5-2, the steel ratios defined below also influence the strength of a beam subjected to combined loading.

$p = A_s/b_o d_o$

$p' = A_s'/b_o d_o$

$p_t = \dfrac{\text{volume of transverse steel}}{\text{gross volume of concrete}}$

Equation (5-1) shows that T_{co} varies linearly with d_o and it is proportional to the square of b_o . Hence the computed torsional strength is more sensitive to variations in b_o than those in d_o . The term T_{s1} is related linearly to the dimensions b' and d'. The number of ties intersected by a potential failure crack may be influenced by concrete covers a_1 , a_2 and a_3. This happens when the dimensions of the steel cage are approximately equal to or a multiple of the spacing of ties. In most cases, however, the concrete covers are almost inconsequential.

The ratio of bottom longitudinal steel, p , governs the term T_{s1} when the flexural tension in this steel is high. The influence of the variation in p , therefore, depends upon the relative magnitude of T_{s1} as compared to the torsional strength. For a beam with a high proportion of transverse steel, the relative magnitude of T_{s1} is generally high. In such a case, a variation in p influences the computed torsional strength significantly. In a case where the flexural tension in the bottom longitudinal steel or the proportion of the transverse steel is very low, the variations in the value of p are not critical.

The ratio of the top longitudinal steel, p' , may govern the torsional strength in a case of hybrid failure. Since such failures are infrequent, the variations in p' are not critical in the computation of torsional strength in most cases.

In majority of cases, the term T_{s1} depends on the ratio of

the transverse steel, p_t . A higher value of p_t generally gives a higher computed torsional strength. However, for a given value of p_t , a closer spacing of ties leads to higher yielding and toughness. The spacing of ties, s , and p_t generally govern the terms T_{s1} and T_{s2} . In the majority of cases, the term T_{s2} is small as compared to T_{s1} . Hence a variation in p_t or the tie spacing would be reflected mostly through a variation in T_{s1} . The influence of a variation in p_t or the tie spacing recedes in importance for a low ratio of the transverse steel.

(iv) Variables Relating to Load Combination

The level of flexural load generally influences every term in the analysis of torsional strength. It has been shown in Section 7-2 that the presence of flexure may increase or decrease the torsional strength. The main effect of flexure is to modify the torque resistance of concrete and to reduce the capacity of the bottom steel to resist the longitudinal component of diagonal tension due to torsion. The level of flexural load in combined bending and torsion has some effect on the area of compressed concrete. It's effect on the lever arm, jd , has been discussed in Section 5-4.

The computed torsional strength is insensitive to variations in the area of compressed concrete for the cleavage type of failure. In this case, even if the flexural compression is assumed to be distributed uniformly over the whole cross-section, the computed torsional strength is not very much affected. The variation in the area of compressed concrete becomes more important in the case of a hybrid failure. The shear (compression) failure of the compressed concrete depends upon the flexural compressive stress, which in turn, depends upon the area of compressed

concrete. The hybrid failure, however, is infrequent. Besides, the contribution of compressed concrete, T_{c1} , is only a minor part of the torsional strength.

A decrease in the lever arm, jd , increases the flexural compressive and tensile forces. The increase in flexural compression increases T_{c1} for the cleavage type of failure, whereas, it decreases T_{c1} for the hybrid type of failure. Since T_{c1} forms only a minor part of the torsional strength as shown above, the variation in flexural compression is not important. The computed torsional strength is more sensitive to the variation in the flexural tension in the bottom longitudinal steel if it inhibits the development of yield stress in the transverse steel due to the lack of adequate resistance to the longitudinal component of diagonal tension. This depends on the relative magnitudes of the steel ratios p and p_t . Whenever the relative magnitude of p_t is small as compared to that of p , the computed torsional strength is insensitive to a variation in the lever arm.

It will be evident from the above discussion that the influences of several variables involved in the analysis of CHAPTER V are interconnected. Hence, quantitative values can not be assigned to the influence of every individual variable. However, since the analysis is simple and direct, it is possible to visualise, at least qualitatively, the influence of any variable which may affect the computed torsional strength of a beam.

7-5 Stress Redistribution

It has been indicated in Section 1-1 that the extent of stress redistribution or the suitability of the elastic and the plastic

theories for computing the torsional strength are controversial topics. In the analysis of CHAPTER V, the extent of redistribution of stresses is somewhat restricted. These restrictions are embodied in Assumptions 5 and 7 of Section 5-2. The characteristics of the concrete and steel strains in this investigation and the observations of Nylander (1945), Gardner (1960) and the author (1963) indicate that the redistribution of stresses is significant. However, its extent is not unrestricted. Hence, the stress condition is not the same at every point of the cross-section at failure. This is evident when the torsional strengths of Beams G-4 and H-4 are compared. These beams, which were tested in pure torsion, had identical transverse steel and equal amounts of longitudinal steel. The only important difference was that the longitudinal steel for Beam G-4 was equally distributed near the top face and the bottom face, whereas, for Beam H-4, it was all concentrated near the bottom face as shown in FIGURE 3-1. The actual torsional strengths of Beams G-4 and H-4 were found to be 115 and 64 in. kips respectively. If there were no restrictions on the extent of stress redistribution, the location of the longitudinal steel would have been inconsequential and consequently Beams G-4 and H-4 would have exhibited equal torsional strength. This, however, is not in conformity with the test evidence cited above.

7-6 Torsional Stiffness

The torsional stiffness of beams subjected to combined bending and torsion has been investigated and is presented in Section 6-7.

Although the reduction in torsional stiffness due to the presence of flexure was first indicated by Chinenkov (1959), no expression for the

torsional stiffness of a beam subjected to combined loading was given by him. Equation (6-14) appears to be the only Equation of its type ever given. In terms of torsional stiffness, which may be defined as the torque required to produce a unit angle of twist per unit length, Equation (6-14) may be expressed as

$$S = S_o/(1 + \frac{M_u}{M_{uo}}) \tag{7-1}$$

where S = initial torsional stiffness of a beam subjected to combined bending and torsion

S_o = initial torsional stiffness of the beam subjected to pure torsion

Both S and S_o refer to the initial slope of the torque-twist curve. As the torque increases, the torsional stiffness reduces slowly so that Equation (7-1) may still be used with some approximation until the cracking torque is reached. Beyond the cracking torque, the torsional stiffness probably is predominantly a function of the geometry of the cross-section and the strength properties of reinforcing steel.

Equation (7-1) may find a useful application in the design of inter-connected bridge girders and other monolithic structures having members in two orthogonal directions. Since the members of such structures are often subjected to combined bending and torsion, the use of Equation (7-1) should result in a better evaluation of the distribution of moments among the inter-connected girders.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

8-1 Summary

The work of this investigation included the following:

(i) Experimental phase

(ii) Analytical phase, and

(iii) Correlation and discussion

The experimental phase of the investigation comprised of tests on 36 beams of 6 x 12-in. nominal size tested in various combinations of bending, torsion and shear. The beams were divided into 8 groups. There were 9 plain concrete beams and 27 beams with both longitudinal bars and ties. Twenty beams with various combinations of steel ratios were tested in combined bending and torsion. The flexural load was applied first and the specimens were then twisted to failure. Sixteen beams were tested in various combinations of bending, torsion and shear. The transverse load was applied first and the specimens were then twisted to failure. Beams in the same group had identical reinforcement but were tested in different combinations of bending, torsion and shear with an objective to obtain interaction diagrams for combined loading. For some beams, most of the longitudinal reinforcement was placed near the bottom face, whereas, for others equal longitudinal steel was provided near the top face and the bottom face. The objective was to study whether the distribution of longitudinal steel had any influence on the shape of the interaction diagram.

From the observations made in the experimental phase of this investigation and utilising the findings of other investigators, an analysis has been developed for predicting the torsional strength of a beam subjected to combined loading. The analysis is direct and general, it's validity extending to the limiting cases.

Using the analysis developed in this investigation, the analytical torsional strengths of 166 beams have been compared with actual torsional strengths. Sixty-eight of these beams were tested by the author and 98 by other investigators. An expression for the initial torsional stiffness of a beam subjected to combined loading has been developed and compared with the test results. The comparisons of torsional strengths and stiffnesses and the discussion of the preceding chapter lead to the conclusions presented in the following section.

8-2 Conclusions

The following are the main conclusions that are drawn from this investigation.

(1) The presence of flexure to a certain limit increases the torsional strength of a typical beam section in which most of the longitudinal steel is located in the zone of flexural tension. This increased strength is essentially due to the greater resistance offered by the portion of concrete compressed due to flexure. The shape of the non-dimensional interaction diagram for the beams in Group D, shown in FIGURE 6-5, is typical for beams in which most of the longitudinal steel is located in the zone of flexural tension. In the case of these beams, the flexural stresses have the effect of a favourable prestress as far as torsional strength is

concerned. The optimum value of flexure, which corresponds to the maximum value of torsional strength, may be evaluated for any particular beam using the analysis presented in this report.

(2) The presence of flexure generally reduces the torsional strength of a beam in which the longitudinal steel is equally distributed in the zones of flexural tension and compression. This reduction is essentially due to the decrease in the capacity of the bottom longitudinal steel to resist the longitudinal component of diagonal tension due to torsion. The shape of the non-dimensional interaction diagram for the beams in Group E, shown in FIGURE 6-5, is typical for beams in which the longitudinal steel is equally distributed in the zones of flexural tension and compression.

(3) A reinforced concrete beam subjected to combined bending and torsion may have one of the following two modes of failure.

(i) A cleavage (tension) failure

(ii) A hybrid failure

The cleavage failure is characterised by the development of diagonal cracks due to torsion on all faces of the specimen.

The hybrid failure is characterised by the shear (compression) failure of the concrete in the zone of flexural compression and cleavage of the concrete not compressed due to flexure.

The cleavage failure is more likely in combined loading though the hybrid failure is possible, especially for high values of the ratio pf_y/f_c' and the bending moment.

The modes of failure predicted by the analysis are in conformity with the observed modes of failure.

(4) The analysis presented in CHAPTER V reproduces the actual trends in the variation of torsional strength as affected by flexure for the two diverse cases of cross-sectional properties discussed in the first two conclusions. This constitutes a confirmation of the concepts used in the analysis. The average value of the ratio of actual and analytical strengths is 1.03 and the mean deviation is 0.09 for all beams tested in combined loading in this investigation.

(5) Values of k_2 equal to zero and unity give, respectively, the lower and the upper bounds of actual torsional strength. It is implied that though the flexural cracks weaken the non-compressed concrete, it is conservative to ignore the non-compressed concrete completely.

(6) The initial torsional stiffness of a reinforced concrete beam is reduced due to the presence of flexure as indicated by Equation (7-1). The average value of the ratio of actual initial torsional stiffness to computed initial torsional stiffness is 0.98 and the mean deviation is 0.14 for all reinforced concrete beams tested in combined bending and torsion in this investigation.

(7) The tests in combined bending, torsion and shear, which were exploratory in nature, suggest that the torsional strength is not reduced due to the presence of transverse shear provided enough transverse steel exists to prevent a shear failure in the absence of torsion.

(8) There is a significant redistribution of stresses as the failure is approached. The extent of redistribution of stresses, however, is not unrestricted. The restrictions embodied in Assumptions 5 and 7 of Section 5-2 are in conformity with test evidence.

CHAPTER IX

RECOMMENDATIONS

9-1 Comments on the Present Status of Codes on Torsion

An extensive survey of the codes of various countries dealing with the requirements of torsion design has been presented by Fisher and Zia (1964). Some of the progressive codes like ACI 318-63, 1963, NBC*, 1960 and CSA, 1959 make no mention of torsion design. The codes, which include instructions for torsion design, with the only exception of the Polish code, are all based on the classical theory of St. Venant. The elastic theory has been utilized even in those instances where an ultimate strength approach is used.

Most of the present building codes, which include torsion design, deal only with the case of pure torsion. The Russian code, Ni Tu 123-55, allows a superposition of the flexural and torsional stresses computed on the assumption that these stresses are independent of each other. Chinenkov's (1959) observations indicate that though this might be satisfactory for low values of the torsional moment, the divergence between analytical and actual strengths becomes large for higher values of the ratio of torsional moment to flexural moment.

9-2 Recommendations for Design

The analysis presented in CHAPTER V may be used in the design of

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* The latest edition of this code, NBC, 1965, is expected to include specifications for torsion design.

members subjected to pure torsion or to combined loading. The analysis uses an ultimate strength approach. Hence, a suitable load factor must be introduced. The value of the load factor may be decided on the same general principles as used in the ultimate strength design of structures, which are not subjected to torsion.

In the computation of torsional strength, the value of k_2 equal to 3/4 gave a close agreement with the actual strengths. However, a comparison of all the tests in combined loading, analysed in TABLES 6-7 through 6-13, indicates that a value of k_2 equal to 1/2 generally gives a safe, yet not very conservative, value of the computed torsional strength.

The design procedure based on the analysis proposed in this investigation may follow the usual steps in design. After selection of a cross-section, the torsional strength may be computed directly if the value of flexural moment is known. If the flexural moment is not known, the interaction diagram for combined bending and torsion may be drawn using the proposed analysis. The value of torsional strength for any level of flexural moment can then be read directly from the interaction diagram.

In the design of monolithic structures involving inter-connected members, the distribution of moments may be determined using the flexural and torsional stiffnesses of the members. The torsional stiffness of members subjected to combined bending and torsion may be determined from Equation (7-1).

Charts and tables, based on the proposed analysis, may serve as useful tools in the design of structures subjected to torsional loads.

9-3 Concluding Remarks

The main objective of this investigation was to study the inadequately explored problem of strength and behavior of reinforced concrete members subjected to combined loading. With this objective tests were carried out on beams with various combinations of reinforcing steel. Based primarily on the observations made during these tests, an analysis for predicting the mode of failure and the torsional strength of beams subjected to combined loading was presented. All tests in combined loading, reported in the literature, which was available, were used to verify the validity of the proposed analysis. It is hoped that the work reported in this investigation will lead to a better understanding of the complex problem of a reinforced concrete member subjected to combined loading.

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APPENDIX A

PROPERTIES OF MATERIALS AND FABRICATION OF TEST SPECIMENS

APPENDIX A

PROPERTIES OF MATERIALS AND FABRICATION OF TEST SPECIMENS

A-1 Materials

(i) Sand

The properties of the sand used are given in TABLES A-1 and A-2. The tests were conducted according to A.S.T.M. Standards, 1961, Part 4.

TABLE A-1

PHYSICAL PROPERTIES OF SAND

Material	Absorption %	Color Test	Specific Gravity		
			Bulk	Apparent	Saturated Surface Dry
Sand	1.36	#2*	2.55	2.63	2.58

*Organic Plate #2

TABLE A-2

SIEVE ANALYSIS OF SAND

Sieve Size	Weight retained (gms)	% retained	Cumulative % retained	A.S.T.M. Standard
#4	10.0	1.6	1.6	0-5
#8	84.0	13.8	15.4	
#16	92.0	15.1	30.5	20-55
#30	127.9	21.0	51.5	
#50	199.6	32.7	84.2	70-90
#100	74.8	12.2	96.4	90-98
Pan	9.6	1.6	--	
Silt	12.3	2.0	--	
Total	610.2	100.0	279.6	
Fineness Modulus 2.80				

The average moisture content of sand was found to be 5.1 percent.

(ii) Coarse Aggregate

The coarse aggregate used was crushed rock of 3/4-inch maximum size. The physical properties of coarse aggregate are given in TABLES A-3 and A-4. The tests were performed according to A.S.T.M. Standards, 1961, part 4.

TABLE A-3

PHYSICAL PROPERTIES OF COARSE AGGREGATE

Material	Absorption %	Dry Rodded Unit Weight lbs/cft	Specific Gravity		
			Bulk	Apparent	Saturated Surface Dry
Crushed rock	1.85	96	2.51	2.61	2.53

TABLE A-4

SIEVE ANALYSIS OF COARSE AGGREGATE

Sieve Size	Weight Retained (lbs)	% retained	Cumulative % retained
3/4 in	3.1	10.5	10.5
3/8 in	19.6	66.5	77.0
#4	6.4	21.8	98.8
Pan	0.3	1.2	100.0
Total	29.4	100.0	286.3

(iii) Reinforcing Steel

All Reinforcing bars were deformed except #2 bars. The load deformation characteristics of the reinforcement were determined by performing coupon tests on at least three specimens in each size. The yield stress and ultimate stress of the reinforcing steel are given in

TABLE A-5 and the stress-strain curves are shown in FIGURE A -1.

TABLE A-5

STRENGTH PROPERTIES OF REINFORCING STEEL

Size of Bar	Average Yield stress ksi	Average Ultimate stress ksi
#2	45.0	58.3
#3	57.3	82.5
#4	58.3	88.4
#5	49.5	75.4
#6	51.3	73.9

(iv) Concrete

The concrete mix design was the same for all the specimens. The materials used for 5 cubic feet of concrete, mixed in the laboratory mixer, were:

(1) Cement 120 lbs.

(2) Sand 310 lbs.

(3) Coarse Aggregate 450 lbs.

High early strength cement, supplied in paper bags, was used. The quantity of water required to produce a mix with about 3-in. slump was of the order of 60 lbs., with minor adjustments to allow for the variation in the moisture content of sand.

Stress-strain curves were obtained from compression tests on only two 6 x 12-in. cylinders (companion cylinders of Beams H-1 and H-3). These stress-strain curves are shown in FIGURE A-2.

A-2 Fabrication of Test Specimens

All concrete was mixed from three to six minutes in a laboratory

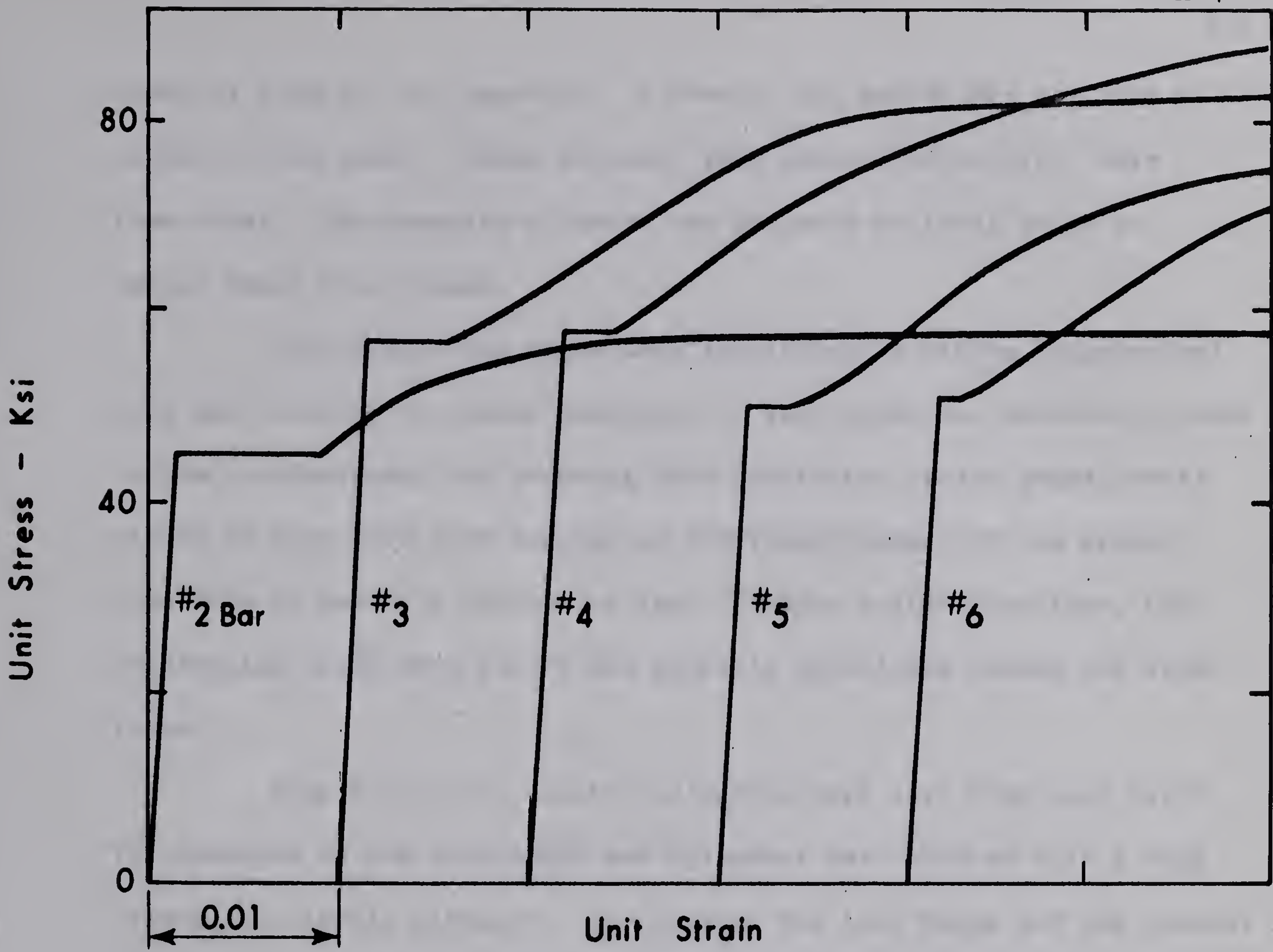


FIGURE A-1 STRESS STRAIN CURVES FOR STEEL

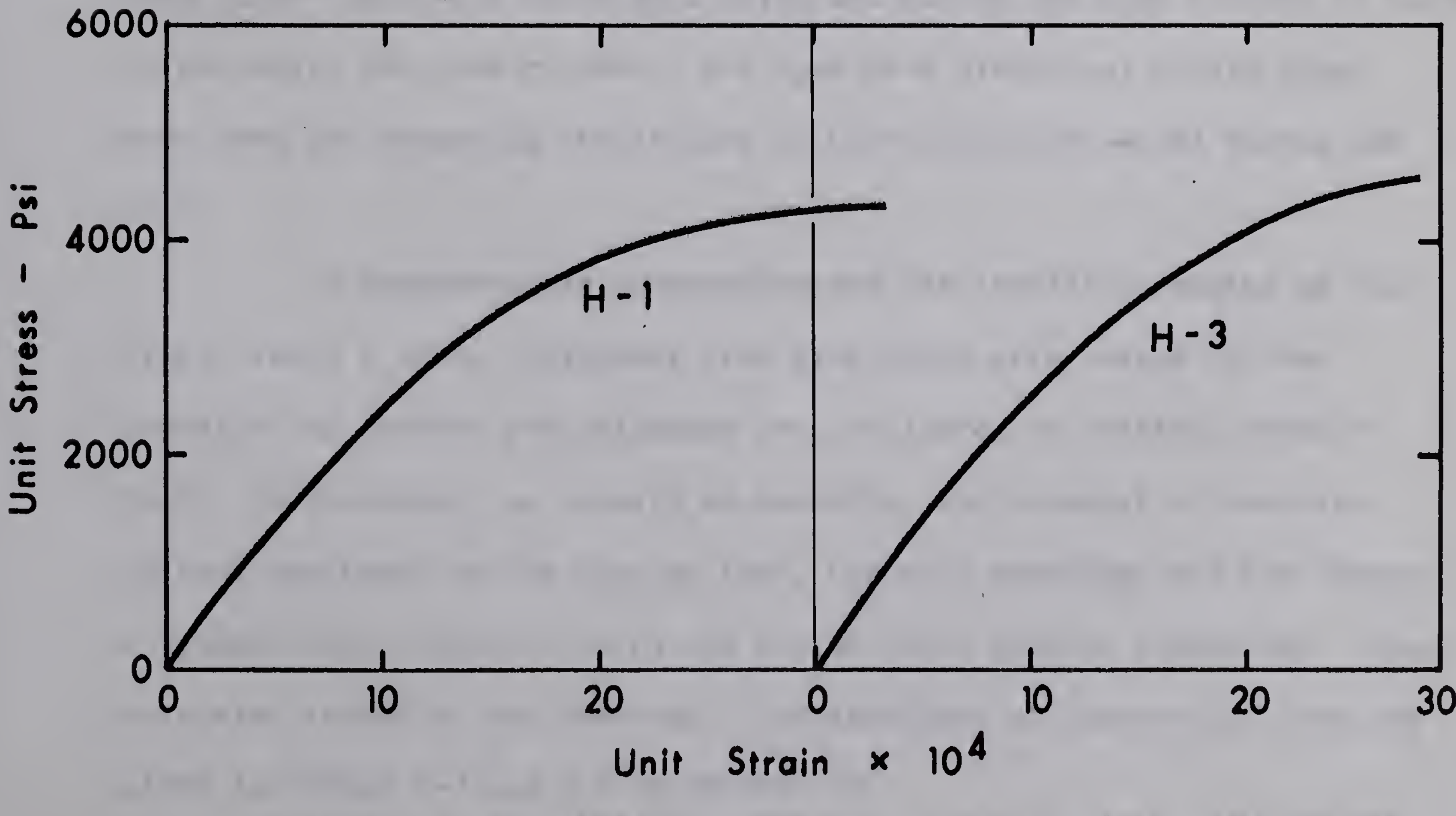


FIGURE A-2 STRESS STRAIN CURVES FOR CONCRETE

FIGURE A-1 STRESS STRAIN CURVES FOR STEEL

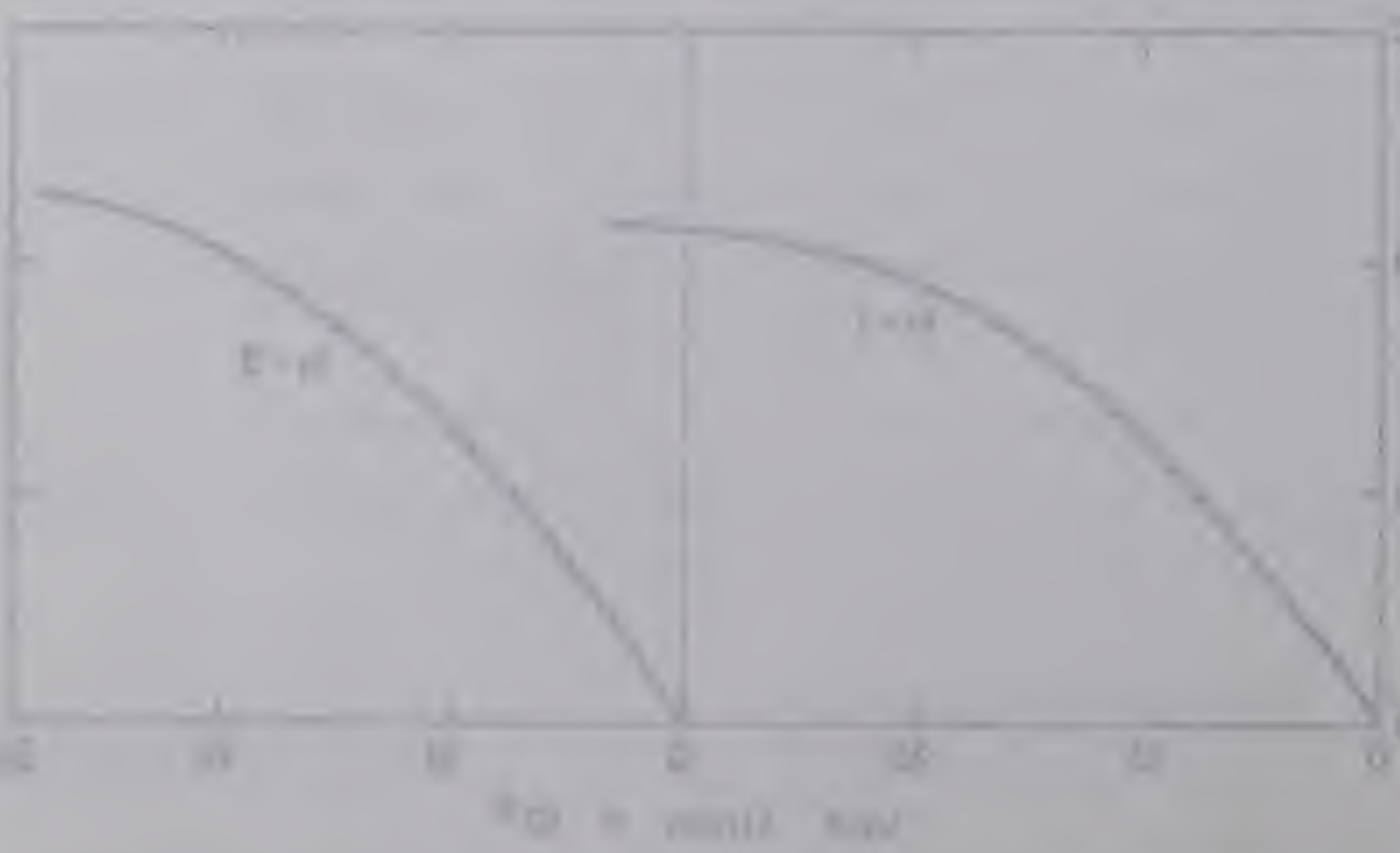


FIGURE A-2 STRESS STRAIN CURVES FOR CONCRETE

mixer of nine cu. ft. capacity. A one-cu. ft. butter mix was used to condition the mixer. Three batches, each about five cu. ft., were then mixed. The quantity of water was adjusted by trial so as to obtain about 3-in. slump.

The reinforcing cages were fabricated by wiring longitudinal bars and stirrups at proper spacing. To form holes for obtaining access to the reinforcement for mounting SR-4 electrical strain gages, small pieces of styrofoam were secured to the reinforcement at the proper locations by means of insulation tape. Before casting concrete, the reinforcing cages were placed and properly positioned inside the steel forms.

Four 6 x 12-in. control cylinders were cast from each batch. The concrete in the test beams and cylinders was vibrated with a high frequency internal vibrator. The tops of the test beams and the control cylinders were trowelled smooth. The test beams and the control cylinders were cured moist for three days using wet burlap and then allowed to cure in air until the time of test. A-7 type SR-4 electrical strain gages were used for measuring the strains in the reinforcing steel during the test.

To determine the compressive and the tensile strengths of concrete, two 6 x 12-in. cylinders from each batch were tested in compression and another two cylinders were subjected to indirect tensile test. To determine, as closely as possible, the strength of concrete in test specimens at the time of test, the test specimens and the control cylinders were vibrated, cured and stored under similar conditions. They were also tested at the same age. The strengths of concrete at test are given in TABLES 4-1 and 4-2 of CHAPTER IV.

APPENDIX B

OBSERVED DATA

APPENDIX B

OBSERVED DATA

B-1 Tests in Combined Bending and Torsion

The observations for each beam tested in combined bending and torsion are given in TABLES B-1 through B-20. The details of the specimens are given in Section 3-1. The testing procedure and the instrumentation of the test specimens are described in Sections 3-3 and 3-4 respectively.

B-2 Tests in Combined Bending, Torsion and Shear

The observations for each beam tested in combined bending, torsion and shear are given in TABLES B-21 through B-36. The details of the specimens are given in Section 3-1. The testing procedure and the instrumentation of the test specimens are described in Sections 3-3 and 3-4 respectively.

B-3 Concrete and Steel Strains

The concrete strains and the steel strains for beams tested in combined bending and torsion are plotted in FIGURES B-1 through B-5. The number affixed to a curve refers to the designation of the location at which the concrete strain was measured. The designations for the locations are shown in FIGURE 3-2.

The concrete strains and the steel strains for beams tested in combined bending, torsion and shear are plotted in FIGURES B-6 through B-9. The locations, at which the concrete strains were measured, are indicated

on the curves. The strains on the two vertical faces of the specimen were not similar due to the effect of transverse shear. Hence the strains on the two vertical faces of the specimen are plotted separately. The designations for the locations are shown in FIGURE 3-3.

The longitudinal strains for beams in Groups A through E are plotted in FIGURE B-10 and those for beams in Groups F through H are plotted in FIGURE B-11. The compressive strain and the tensile strain are plotted respectively on the left side and the right side of the zero line. The number affixed to a curve refers to the loading stage at which the strains were measured. The loading stages are shown in TABLE B-1 through B-36. The transverse shear produced dissimilar strain conditions on the two vertical faces of the specimen. Hence, for the specimens tested in combined bending, torsion and shear, strains on both vertical faces are plotted. The letters N and S affixed to the curves refer to the North face and the South face respectively. The shearing stresses due to torsion and transverse shear were of the same sense on the South face and they were of the opposite sense on the North face.

B-4 Deformation Characteristics

The deformation characteristics for the beams tested in combined bending and torsion are shown in FIGURE B-12. The positions of the dial gages for the measurement of deflections and the positions of the twist-meters for the measurement of angle of twist are shown in FIGURE 3-2. The deflections of the mid-point of the beam relative to the ends of the beam are plotted in FIGURE B-12.

The deformation characteristics for the beams tested in combined bending, torsion and shear are shown in FIGURE B-13. The positions of the

dial gages for the measurement of deflections and the positions of the twistmeters for the measurement of angle of twist are shown in FIGURE 3-3. The numbers affixed to the curves for deflections indicate the dial gage positions shown in FIGURE 3-3.

TABLE B-1

PURE BENDING TEST ON BEAM A-1

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Remarks
1	0	0	0	
2	0.5	25	0	
3	1.0	33	0.5	
4	1.5	42	1.5	
5	2.0	50	1.5	
6	2.5	59	2.5	Sudden Failure

TABLE B-2

COMBINED BENDING AND TORSION TEST ON BEAM A-2

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Remarks
1	0	0	0	0	--	
2	0.5	33	0	0	--	
3	1.0	42	1	0	--	
4	1.0	42	-	8	4.5	
5	1.0	42	-	12	8.9	
6	1.0	42	-	18	11.1	
7	1.0	42	-	23	15.7	
8	1.0	42	-	28	22.2	
9	1.0	42	-	33	24.4	
10	1.0	42	-	38	33.3	
11	1.0	42	-	53	--	Sudden Failure

TABLE B-3

COMBINED BENDING AND TORSION TEST ON BEAM A-3

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Remarks
1	0	0	0	0	--	
2	0.3	30	1	0	--	
3	0.6	35	1	0	--	
4	0.6	35	-	12	11.1	
5	0.6	35	-	23	22.2	
6	0.6	35	-	33	35.6	
7	0.6	35	-	38	46.6	
8	0.6	35	-	43	51.1	
9	0.6	35	-	49	66.6	
10	0.6	35	-	53	--	Sudden Failure

TABLE B-4

COMBINED BENDING AND TORSION TEST ON BEAM A-4

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radian per inch x 10^6	Remarks
1	0	14	0	0	--	
2	0	25	1	0	--	
3	-	25	-	12	11.1	
4	-	25	-	23	22.2	
5	-	25	-	33	37.8	
6	-	25	-	43	57.8	
7	-	25	-	52	--	Sudden Failure

TABLE B-5

PURE TORSION TEST ON BEAM A-5

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Remarks
1	0	0	
2	12	13.3	
3	23	26.7	
4	33	42.3	
5	38	48.9	
6	43	57.8	
7	49	71.1	
8	54	91.1	
9	58	--	Sudden Failure

TABLE B-6

PURE BENDING TEST ON BEAM B-1

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Steel stresses(ksi) Bottom Bar	Tie	Remarks
1	0	0	0	0	0	
2	2	50	4	3.2	2.1	
3	4	84	9	14.7	3.5	
4	6	118	14	26.6	4.7	
5	7	135	15	31.5	5.0	
6	8	152	18	36.6	5.1	
7	9	169	21	42.0	5.3	
8	10	186	22	47.3	5.4	
9	12	220	27	Yield	5.6	Wide cracks
Test discontinued						

TABLE B-7

COMBINED BENDING AND TORSION TEST ON BEAM B-2

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel stresses(ksi) Bottom Bar	Tie
1	0	0	0	0	--	0	0
2	2	59	5	0	--	8.3	2.7
3	4	93	11	0	--	21.9	3.9
4	6	127	16	0	--	34.5	4.2
5	8	161	23	0	--	45.9	4.5
6	10	195	31	0	--	Yield	4.5
7	10	195	--	23	55.5	Yield	4.1
8	10	195	--	43	167	Yield	22.2
9*	10	195	--	64	910	Yield	51.2
10**	10	195	--	72	--	Yield	-

* Torsion cracks on top face
** Failure

TABLE B-8

COMBINED BENDING AND TORSION TEST ON BEAM B-3

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)	
						Bottom Bar	Tie
1	0	0	0	0	--	0	0
2	2	59	4	0	--	4.2	2.1
3	4	93	12	0	--	14.7	4.2
4	5	110	16	0	--	19.2	4.4
5	5	110	--	12	15.6	22.5	4.4
6	5	110	--	23	40.0	23.7	4.4
7	5	110	--	33	66.7	25.2	4.5
8	5	110	--	43	107	27.5	4.8
9*	5	110	--	54	171	33.8	5.4
10**	5	110	--	64	405	37.5	8.7
11	5	110	--	74	855	Yield	18.9
12	5	110	--	85	1255	--	26.4
13***	5	110	--	95	--	--	Yield

* Torsion cracks on sides
** Torsion cracks on top face
*** Failure

TABLE B-9

PURE TORSION TEST ON BEAM B-4

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)			Remarks
			Bottom Bar	Tie	Top Bar	
1	0	0	0	0	0	
2	12	17.8	0.2	0	0	
3	23	33.3	0.2	0	0.2	
4	33	51.2	0.6	0	0.3	
5	43	80.0	1.1	0	0.8	
6	54	504	28.2	6.2	40.8	Cracking
7	64	700	35.4	13.8	50.4	
8	69	898	40.5	20.1	55.8	
9	74	1070	46.5	24.0	Yield	
10	80	1305	53.0	30.0	Yield	
11	85	--	Yield	31.5	Yield	Failure

TABLE B-10

COMBINED BENDING AND TORSION TEST ON BEAM C-1

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	3	76	7	0	--	13.8	2.0	-2.6
3	6	127	16	0	--	27.0	3.2	-4.7
4	9	178	23	0	--	38.6	4.4	-6.6
5	12	229	29	0	--	49.8	5.1	-8.9
6*	15	280	36	0	--	Yield	5.1	-15.2
7	15	280	--	12	15.5	Yield	5.6	-15.5
8	15	280	--	23	53.3	Yield	6.2	-15.8
9	15	280	--	33	118	Yield	7.1	-15.5
10	15	280	--	43	189	Yield	8.6	-15.3
11	15	280	--	54	264	Yield	11.1	-14.7
12	15	280	--	64	443	Yield	13.8	-13.8
13	15	280	--	71	--	Yield	18.0	-11.0
14	15	280	--	74	--	Yield	19.8	-9.0
15**	15	280	--	78	--	Yield	--	--

* Wide flexural cracks
** Failure

TABLE B-11

COMBINED BENDING AND TORSION TEST ON BEAM C-2

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	2.5	67	4	0	--	10.5	2.7	-4.5
3	5.0	110	9	0	--	20.9	3.9	-8.7
4	7.5	152	15	0	--	30.2	5.1	-12.5
5*	10.0	195	16	0	--	39.0	5.6	-15.6
6	10.0	195	--	23	40.0	41.0	5.7	-16.7
7	10.0	195	--	43	102	41.6	6.0	-16.4
8	10.0	195	--	64	233	43.2	8.1	-15.0
9	10.0	195	--	74	356	45.3	10.2	-11.9
10**	10.0	195	--	85	538	49.2	13.1	-6.3
11	10.0	195	--	95	1035	Yield	27.0	+11.0
12***	10.0	195	--	105	--	Yield	54.2	+21.8

* Fair sized flexural cracks
** Torsion cracks on top face
*** Failure

TABLE B-12

COMBINED BENDING AND TORSION TEST ON BEAM C-3

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)		
						Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	2	59	2	0	--	3.3	0	-2.3
3	3	76	5	0	--	6.6	0.5	-4.1
4	4	93	7	0	--	13.5	1.1	-5.6
5	5	110	9	0	--	20.1	1.4	-8.1
6	5	110	-	22	31.2	22.2	1.7	-8.6
7*	5	110	-	43	111	25.1	2.1	-7.8
8	5	110	-	53	171	28.4	3.3	-5.9
9	5	110	-	64	327	30.3	5.0	-1.8
10**	5	110	-	74	547	35.7	8.4	+4.1
11	5	110	-	84	887	44.7	11.1	+12.8
12	5	110	-	95	1145	49.5	13.1	+18.2
13	5	110	-	105	1545	Yield	14.7	+24.3
14	5	110	-	110	--	Yield	15.5	+32.3
15***	5	110	-	111	--	Yield	--	--

* Torsion cracks on sides
** Torsion cracks on top face
*** Failure

TABLE B-13

PURE TORSION TEST ON BEAM C-4

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)			Remarks
			Bottom Bar	Tie	Top Bar	
1	0	0	0	0	0	
2	22	24.4	0.6	0.5	0.2	
3	43	51.1	1.5	1.5	0.8	
4	53	84.5	3.6	2.7	3.2	Cracks on sides
5	64	272	16.5	9.6	26.7	Cracks on top
6	74	518	19.2	14.7	41.3	
7	84	690	20.7	18.5	49.7	
8	95	960	22.8	23.3	Yield	
9	105	1190	26.1	28.7	Yield	
10	111	--	36.0	44.0	Yield	Failure

TABLE B-14

COMBINED BENDING AND TORSION TEST ON BEAM D-1

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)		
						Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	5	110	4	0	--	7.8	1.2	-5.3
3	10	195	14	0	--	16.5	1.8	-10.5
4	20	365	29	0	--	33.2	3.6	-20.4
5	30	535	34	0	--	49.5	4.7	-30.8
6	36	637	--	0	--	Yield	4.2	Yield
7	36	637	--	22	80.0	Yield	4.5	Yield
8*	36	637	--	43	211	Yield	5.1	Yield
9	36	637	--	64	522	Yield	6.9	Yield
10**	36	637	--	84	1530	Yield	24.0	Yield
11***	36	637	--	99	--	Yield	55.8	Yield

* Crushing on top face
** Cracking and spalling on top face
*** Failure

TABLE B-15

COMBINED BENDING AND TORSION TEST ON BEAM D-2

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)	
						Tie	Top Bar
1	0	0	0	0	--	0	0
2	5	110	5	0	--	0.8	-4.7
3	10	195	8	0	--	1.4	-10.1
4	15	280	13	0	--	1.8	-15.5
5	20	365	17	0	--	2.0	-23.0
6	20	365	--	22	26.7	1.7	-23.7
7	20	365	--	43	69.0	1.7	-24.2
8	20	365	--	64	138	1.8	-24.2
9*	20	365	--	84	318	4.4	-22.7
10**	20	365	--	105	567	8.3	-18.8
11	20	365	--	126	938	14.6	-12.5
12	20	365	--	136	1130	19.5	-10.8
13	20	365	--	146	1395	24.5	-8.4
14	20	365	--	157	1755	30.8	-5.6
15***	20	365	--	164	--	47.0	+8.9

* Torsion cracks on sides
** Torsion cracks on top face
*** Failure

TABLE B-16

COMBINED BENDING AND TORSION TEST ON BEAM D-3

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	2	59	3	0	--	2.1	0.5	-2.6
3	4	93	3	0	--	5.7	1.1	-5.4
4	6	127	8	0	--	9.6	1.7	-8.4
5	8	161	12	0	--	12.3	2.3	-11.1
6	10	195	15	0	--	15.8	2.7	-14.3
7	10	195	--	22	35.6	16.2	2.3	-14.6
8	10	195	--	43	97.7	17.1	1.5	-14.4
9*	10	195	--	64	256	19.8	3.3	-10.2
10	10	195	--	84	550	22.8	9.5	-1.7
11**	10	195	--	105	910	24.6	20.0	+7.7
12	10	195	--	126	1280	27.3	31.8	+17.0
13	10	195	--	146	1860	27.3	44.9	+29.7
14***	10	195	--	156	--	36.3	Yield	+47.4

* Torsion cracks on sides
** Torsion cracks on top face
*** Failure

TABLE B-17

PURE TORSION TEST ON BEAM D-4

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar	Remarks
1	0	0	0	0	0	
2	22	28.8	0.5	0.8	0.5	
3	43	104.5	3.2	2.7	2.1	Cracks on sides
4	64	396	9.2	4.8	21.5	Cracks on top face
5	84	690	12.8	7.5	30.5	
6	105	1040	15.6	10.4	39.8	
7	126	1435	18.6	13.8	49.5	
8	136	1730	20.4	16.4	Yield	
9	146	2170	23.9	40.8	Yield	Failure

TABLE B-18

COMBINED BENDING AND TORSION TEST ON BEAM E-1

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)		
						Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	2	59	5	0	--	13.8	2.9	-4.2
3	4	93	11	0	--	27.9	4.4	-7.5
4	6	127	17	0	--	39.3	4.5	-10.5
5	8	161	22	0	--	50.0	4.7	-13.5
6	10	195	28	0	--	Yield	5.0	-18.0
7	10	195	--	22	53.2	Yield	5.0	-17.0
8*	10	195	--	43	160	Yield	5.0	-15.2
9	10	195	--	53	262	Yield	5.4	-14.1
10	10	195	--	64	532	Yield	16.8	-7.5
11	10	195	--	74	--	Yield	45.3	-3.9
12**	10	195	--	76	--	Yield	Yield	--

* Torsion cracks on sides
** Torsion cracks on top face and failure

TABLE B-19

COMBINED BENDING AND TORSION TEST ON BEAM E-2

Loading Stage	Bending Load kips	Bending Moment in kips	Deflection in x 10^3	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)		
						Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	2	59	3	0	--	--	1.1	-3.8
3	3	76	9	0	--	20.9	2.9	-7.1
4	4	93	12	0	--	24.6	3.0	-9.2
5	5	110	16	0	--	30.6	3.2	-11.0
6	5	110	--	22	35.6	32.4	3.5	-11.4
7*	5	110	--	43	131	34.8	3.8	-9.2
8	5	110	--	53	229	37.2	4.4	-4.1
9**	5	110	--	64	412	42.2	6.8	+4.7
10	5	110	--	74	667	45.9	12.3	+15.0
11	5	110	--	84	1005	52.8	19.4	+25.8
12	5	110	--	95	1425	54.6	27.5	+36.0
13	5	110	--	101	--	57.3	36.5	--

* Torsion cracks on sides
** Torsion cracks on top face

TABLE B-20

PURE TORSION TEST ON BEAM E-3

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar	Remarks
1	0	0	0	0	0	
2	22	28.8	0	0	0	
3	43	80.0	1.5	0	0.8	
4	53	332	10.2	6.2	8.7	Cracks on sides and top
5	64	585	16.4	9.8	15.3	
6	74	810	21.6	13.1	18.5	
7	84	1065	25.8	16.4	21.0	
8	95	1325	30.0	20.1	23.4	
9	105	1590	34.2	24.3	24.3	
10	115	2080	44.7	32.7	26.1	
11	121	--	Yield	--	--	Failure

TABLE B-21

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM A-6

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Remarks
1	0	0	0	0	--	
2	0.5	14	0.6	0	--	
3	1.0	19	0.9	0	--	
4	1.5	24	1.1	0	--	
5	2.0	29	1.4	0	--	
6	2.5	33	1.6	0	--	
7	2.5	33	1.6	12	11.1	
8	2.5	33	1.6	22	22.2	
9	2.5	33	1.6	33	37.1	
10	2.5	33	1.6	43	51.8	
11	2.5	33	1.6	52	--	Sudden Failure

TABLE B-22

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM A-7

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Remarks
1	0	0	0	0	--	
2	1.0	19	0.9	0	--	
3	1.5	24	1.1	0	--	
4	2.0	29	1.4	0	--	
5	2.5	33	1.6	0	--	
6	3.0	38	1.9	0	--	
7	3.3	41	2.0	0	--	
8	3.3	41	2.0	12	7.4	
9	3.3	41	2.0	22	25.9	
10	3.3	41	2.0	33	148	
11	3.3	41	2.0	40	--	Sudden Failure

TABLE B-23

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM A-8

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Remarks
1	0	0	0	0	--	
2	0.5	14	0.6	0	--	
3	1.0	19	0.9	0	--	
4	1.5	24	1.1	0	--	
5	1.5	24	1.1	12	14.8	
6	1.5	24	1.1	22	29.6	
7	1.5	24	1.1	33	51.8	
8	1.5	24	1.1	43	--	Sudden Failure

TABLE B-24

PURE TORSION TEST ON BEAM A-9

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Remarks
1	0	0	
2	12	14.8	
3	22	29.6	
4	33	59.2	
5	42	--	Sudden Failure

TABLE B-25

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM F-1

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	3	27	1.7	0	--	0.9	0	-0.9
3	6	50	3.2	0	--	2.3	2.4	-1.8
4	9	72	4.7	0	--	9.8	7.5	-2.4
5	12	95	6.2	0	--	26.7	14.4	-2.4
6	14	110	7.2	0	--	33.3	18.0	-2.9
7	16	125	8.2	0	--	39.3	22.4	-3.3
8*	18	140	9.2	0	--	49.2	24.9	-4.4
9	18	140	9.2	12	66.7	50.3	25.2	-3.6
10	18	140	9.2	22	150	49.8	28.2	-3.3
11	18	140	9.2	33	267	49.5	34.2	-3.0
12**	18	140	9.2	43	521	48.3	41.4	-2.9
13	18	140	9.2	48	876	50.7	52.2	-2.4
14***	18	140	9.2	58	--	Yield	Yield	+6.3

* Wide flexure shear cracks
** Torsion cracks on top face
*** Failure

TABLE B-26

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM F-2

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	2	20	1.2	0	--	0.6	0	-0.5
3	4	35	2.2	0	--	1.4	0.5	-1.1
4	6	50	3.2	0	--	1.8	2.0	-1.8
5	8	65	4.2	0	--	4.4	3.9	-2.4
6	10	80	5.2	0	--	14.1	6.5	-2.9
7	12	95	6.2	0	--	23.4	7.2	-3.6
8	12	95	6.2	12	20.8	23.4	7.5	-3.5
9	12	95	6.2	22	45.8	23.4	8.1	-2.9
10*	12	95	6.2	33	108	24.6	8.6	+0.8
11	12	95	6.2	43	172	26.3	9.0	+3.9
12**	12	95	6.2	53	312	29.0	15.6	+8.7
13***	12	95	6.2	64	458	32.3	24.2	+19.4
14	12	95	6.2	74	815	48.5	Yield	+27.6
15****	12	95	6.2	83	--	Yield	Yield	+52.2

* Torsion cracks on south face
** Torsion cracks on top face
*** Torsion cracks on north face
**** Failure

TABLE B-27

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM F-3

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)		
						Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	2	20	1.2	0	--	0.8	0	-0.8
3	4	35	2.2	0	--	1.8	0.2	-1.5
4	5	42	2.7	0	--	2.4	0.9	-2.1
5	6	50	3.2	0	--	5.0	2.4	-3.8
6	6	50	3.2	12	20.8	4.8	2.4	-3.3
7	6	50	3.2	22	54.2	4.8	2.7	-3.0
8*	6	50	3.2	33	100	10.8	13.1	-1.7
9	6	50	3.2	43	150	13.5	18.3	-0.6
10**	6	50	3.2	53	375	19.5	25.5	+28.5
11***	6	50	3.2	64	563	33.6	36.0	+39.6
12	6	50	3.2	74	870	51.0	52.5	+53.3
13	6	50	3.2	84	1545	Yield	Yield	Yield
14	6	50	3.2	89	--	Yield	Yield	Yield

* Torsion cracks on south face
** Torsion cracks on top face
*** Torsion cracks on north face

TABLE B-28

PURE TORSION TEST ON BEAM F-4

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)			Remarks
			Bottom Bar	Tie	Top	
1	0	0	0	0	0	
2	12	8.3	0	0.3	0	
3	22	20.8	0	0.5	0	
4	33	37.5	0.5	0.9	0	
5	43	75.0	1.2	2.0	0	Cracks on sides
6	53	425	35.7	34.8	5.6	Cracks on top face
7	64	675	44.4	45.9	15.5	
8	74	1160	52.5	Yield	35.3	
9	84	1540	Yield	Yield	45.3	
10	95	3330	Yield	Yield	Yield	Failure

TABLE B-29

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM G-1

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)		
						Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	4	35	2.2	0	--	1.8	0	-1.8
3	8	65	4.2	0	--	18.9	5.9	-4.2
4	12	95	6.2	0	--	32.4	7.5	-4.5
5	16	125	8.2	0	--	46.2	8.7	-5.3
6	18	140	9.2	0	--	55.4	9.9	-5.7
7	19	147	9.7	0	--	Yield	10.5	-5.9
8	20	155	10.2	0	--	Yield	12.0	-5.4
9	20	155	10.2	12	41.6	Yield	15.2	-3.9
10	20	155	10.2	22	133	Yield	18.0	-2.3
11	20	155	10.2	33	225	Yield	21.9	-1.2
12	20	155	10.2	43	425	Yield	27.8	+17.1
13*	20	155	10.2	53	680	Yield	32.6	+25.2
14**	20	155	10.2	64	1520	Yield	44.9	+33.6
15***	20	155	10.2	73	--	Yield	Yield	+37.2

* Torsion cracks on north face
** Torsion cracks on top face
*** Failure

TABLE B-30

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM G-2

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi)		
						Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	3	27	1.7	0	--	1.5	0	-1.1
3	6	50	3.2	0	--	5.6	0.9	-2.6
4	9	72	4.7	0	--	23.0	5.6	-3.6
5*	12	95	6.2	0	--	35.7	8.1	-4.2
6	12	95	6.2	12	33.3	35.6	9.9	-3.6
7**	12	95	6.2	22	66.7	33.3	10.8	-1.5
8***	12	95	6.2	33	117	32.9	12.0	0
9	12	95	6.2	43	175	34.1	15.5	+6.0
10****	12	95	6.2	53	322	36.6	19.4	+15.9
11*****	12	95	6.2	64	613	51.3	Yield	+24.6
12	12	95	6.2	74	865	Yield	Yield	+33.6
13	12	95	6.2	84	1480	Yield	Yield	+45.0
14******	12	95	6.2	92	--	Yield	Yield	+53.1

* Flexure shear cracks upto about 3 inches from top face
** Extension of shear cracks on south face due to torsion
*** Torsion cracks on south face
**** Torsion cracks on top face
***** Torsion cracks on north face
****** Failure

TABLE B-31

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM G-3

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar
1	0	0	0	0	--	0	0	0
2	2	20	1.2	0	--	0.9	0	-0.6
3	4	35	2.2	0	--	1.8	0	-1.5
4	5	42	2.7	0	--	2.4	0	-1.8
5*	6	50	3.2	0	--	3.2	0	-2.4
6**	6	50	3.2	12	22.8	3.5	0.3	-2.4
7	6	50	3.2	22	50.0	6.8	0.6	-0.3
8	6	50	3.2	33	113	8.9	13.2	+0.9
9	6	50	3.2	43	163	10.4	19.5	+1.8
10	6	50	3.2	53	246	12.2	25.2	+4.1
11***	6	50	3.2	64	517	20.9	30.0	+32.6
12	6	50	3.2	74	672	29.7	37.8	+42.0
13	6	50	3.2	84	1090	44.0	Yield	+51.6
14	6	50	3.2	95	1505	Yield	Yield	Yield
15****	6	50	3.2	103	--	Yield	Yield	Yield

* Flexure shear cracks upto about 5 inches from top face
** Torsion cracks as extension of shear cracks on south face
*** Torsion cracks on north and top faces
**** Failure

TABLE B-32

PURE TORSION TEST ON BEAM G-4

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Tie	Top Bar	Remarks
1	0	0	0	0	0	
2	12	8.3	0	0.2	0	
3	22	20.8	0	0.5	0.3	
4	33	45.8	0	0.9	0.6	
5	43	87.5	0.5	2.0	0.9	Cracking torque = 50 k.in.
6	53	212	7.2	22.2	5.3	Cracks on sides and top
7	64	462	18.3	34.5	27.0	
8	74	745	26.4	Yield	41.4	
9	84	1020	36.3	Yield	52.8	
10	95	1360	45.2	Yield	Yield	
11	105	1710	54.3	Yield	Yield	
12	115	2680	Yield	Yield	Yield	Failure

TABLE B-33

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM H-1

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Steel Stresses (ksi) Tie
1	0	0	0	0	--	0	0
2	4	35	2.2	0	--	2.1	-0.3
3	8	65	4.2	0	--	9.6	-0.3
4	12	95	6.2	0	--	17.1	-0.8
5	16	125	8.2	0	--	26.9	-1.8
6	20	155	10.2	0	--	36.0	-0.6
7	24	185	12.2	0	--	44.7	+3.3
8	28	215	14.2	0	--	48.9	+8.4
9	30	230	15.2	0	--	Yield	+7.2
10*	32	245	16.2	0	--	Yield	Yield
11	32	245	16.2	12	33.3	Yield	Yield
12	32	245	16.2	22	91.7	Yield	Yield
13	32	245	16.2	33	171	Yield	Yield
14	32	245	16.2	43	278	Yield	Yield
15	32	245	16.2	53	455	Yield	Yield
16**	32	245	16.2	64	1020	Yield	Yield
17***	32	245	16.2	74	--	Yield	Yield

* Wide flexure shear cracks
** Some visible crushing on top face
*** Failure

TABLE B-34

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM H-2

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Steel Stresses (ksi) Tie
1	0	0	0	0	--	0	0
2	4	35	2.2	0	--	2.7	0.6
3	8	65	4.2	0	--	8.9	2.7
4	12	95	6.2	0	--	16.5	4.5
5	16	125	8.2	0	--	24.6	5.7
6	20	155	10.2	0	--	37.2	11.4
7	20	155	10.2	12	41.6	38.7	12.3
8*	20	155	10.2	22	96.0	40.4	13.8
9	20	155	10.2	33	159	42.0	17.1
10**	20	155	10.2	43	246	44.1	21.5
11***	20	155	10.2	53	379	47.3	28.1
12	20	155	10.2	64	600	Yield	37.5
13****	20	155	10.2	74	975	Yield	Yield
14*****	20	155	10.2	84	--	Yield	Yield

* Extension of shear cracks and new torsion cracks on south face
** Torsion cracks on top face over about middle third width
*** Torsion cracks on top face extended to south edge of beam
**** Torsion cracks on top face over entire width
***** Failure

TABLE B-35

COMBINED BENDING SHEAR AND TORSION TEST ON BEAM H-3

Loading Stage	Bending Load kips	Average Bending Moment in kips	Shear Force kips	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Steel Stresses (ksi) Tie
1	0	0	0	0	--	0	0
2	2	20	1.2	0	--	0.6	0
3	4	35	2.2	0	--	1.8	0.3
4	6	50	3.2	0	--	4.1	1.4
5	8	65	4.2	0	--	8.0	3.6
6	10	80	5.2	0	--	12.3	4.8
7	10	80	5.2	12	8.3	13.2	4.8
8*	10	80	5.2	22	33.3	14.7	4.8
9	10	80	5.2	33	66.7	15.6	5.3
10	10	80	5.2	43	104	18.0	5.9
11	10	80	5.2	53	171	23.1	11.0
12**	10	80	5.2	64	413	33.9	28.2
13	10	80	5.2	74	613	39.9	35.9
14	10	80	5.2	84	982	47.1	42.6
15***	10	80	5.2	88	--	--	--

* Torsion cracks on south face
** Torsion cracks on north and top faces
*** Failure

TABLE B-36

PURE TORSION TEST ON BEAM H-4

Loading Stage	Torque in kips	Twist Radians per inch x 10^6	Steel Stresses (ksi) Bottom Bar	Steel Stresses (ksi) Tie	Remarks
1	0	0	0	0	
2	12	4.2	0	0	
3	22	16.7	0	0	
4	33	33.3	0	0	
5	43	54.2	0.6	0.6	Cracking Torque = 51 k.in.
6	53	108	6.3	11.3	Cracks on sides
7	64	--	9.9	39.6	Sudden Failure

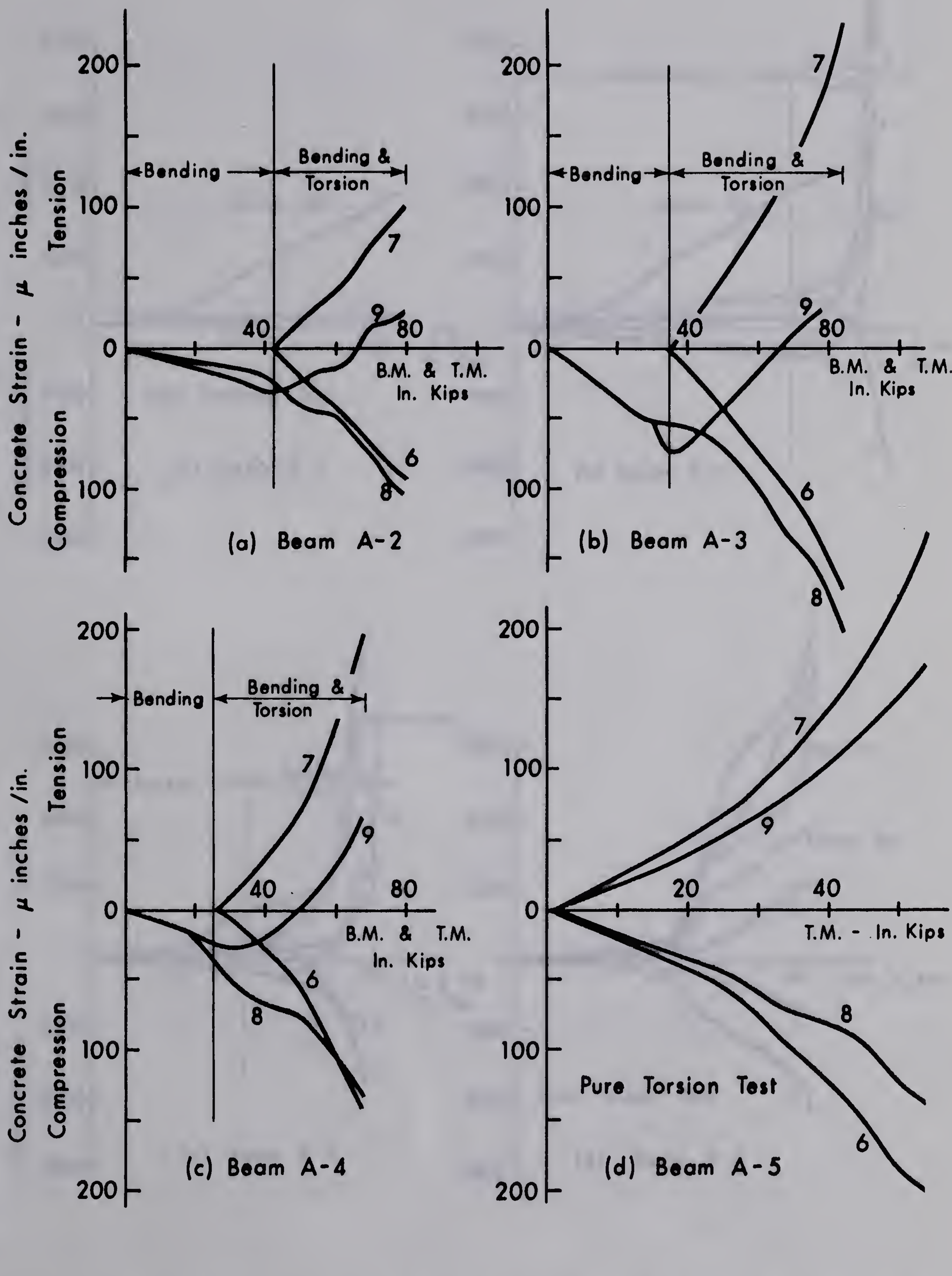


FIGURE B-1 CONCRETE STRAINS FOR BEAMS IN GROUP A

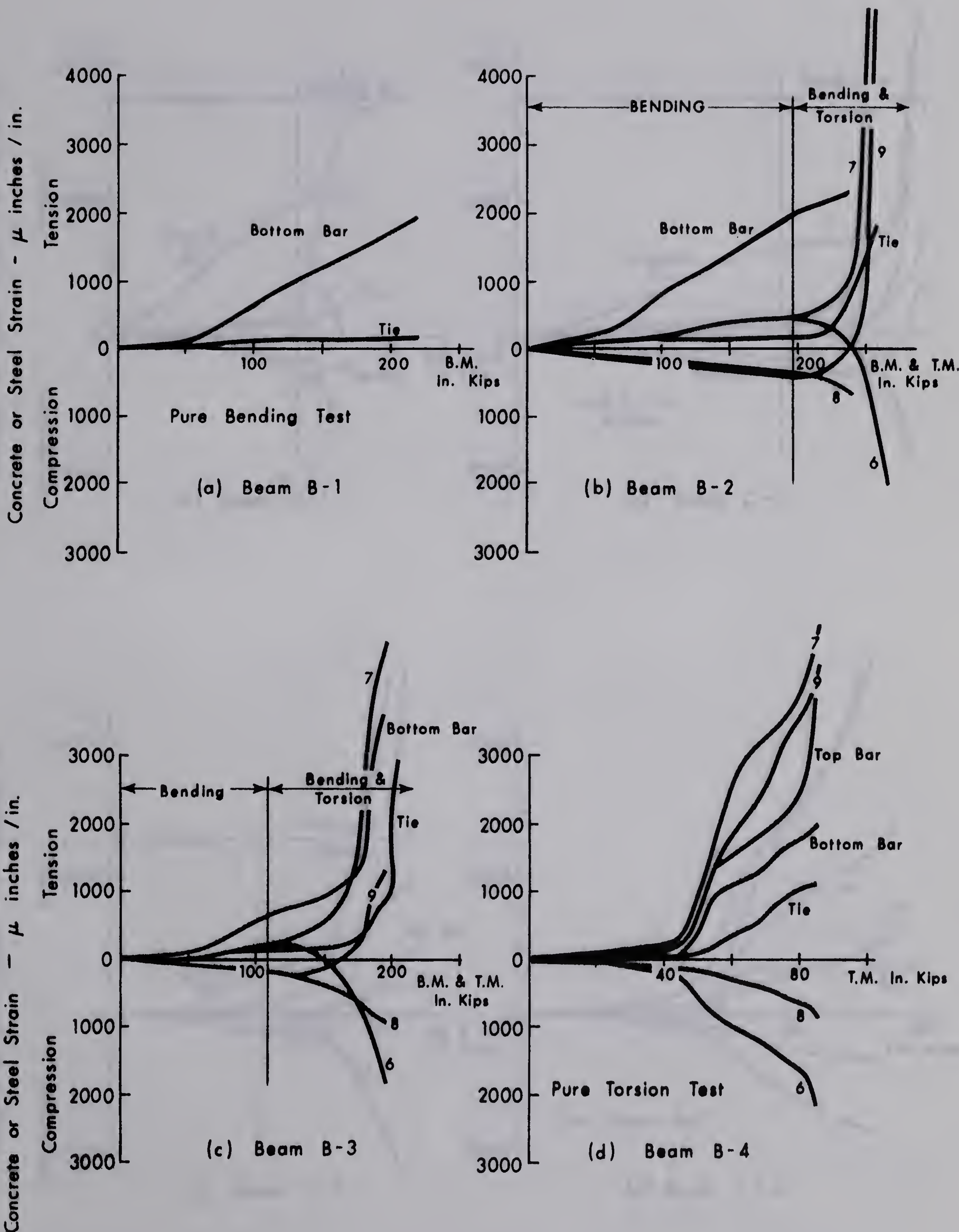


FIGURE B-2 CONCRETE & STEEL STRAINS FOR BEAMS IN GROUP B

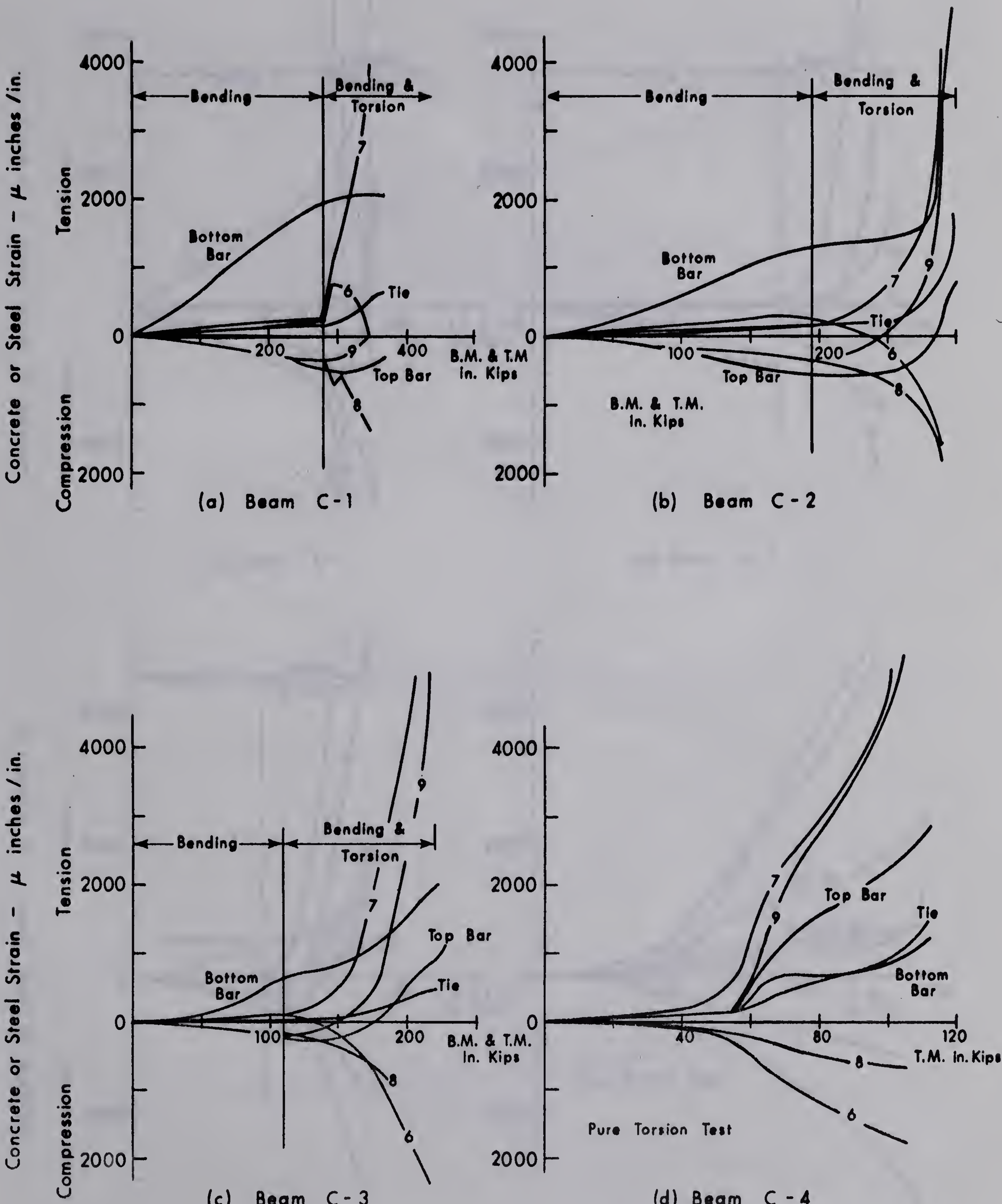
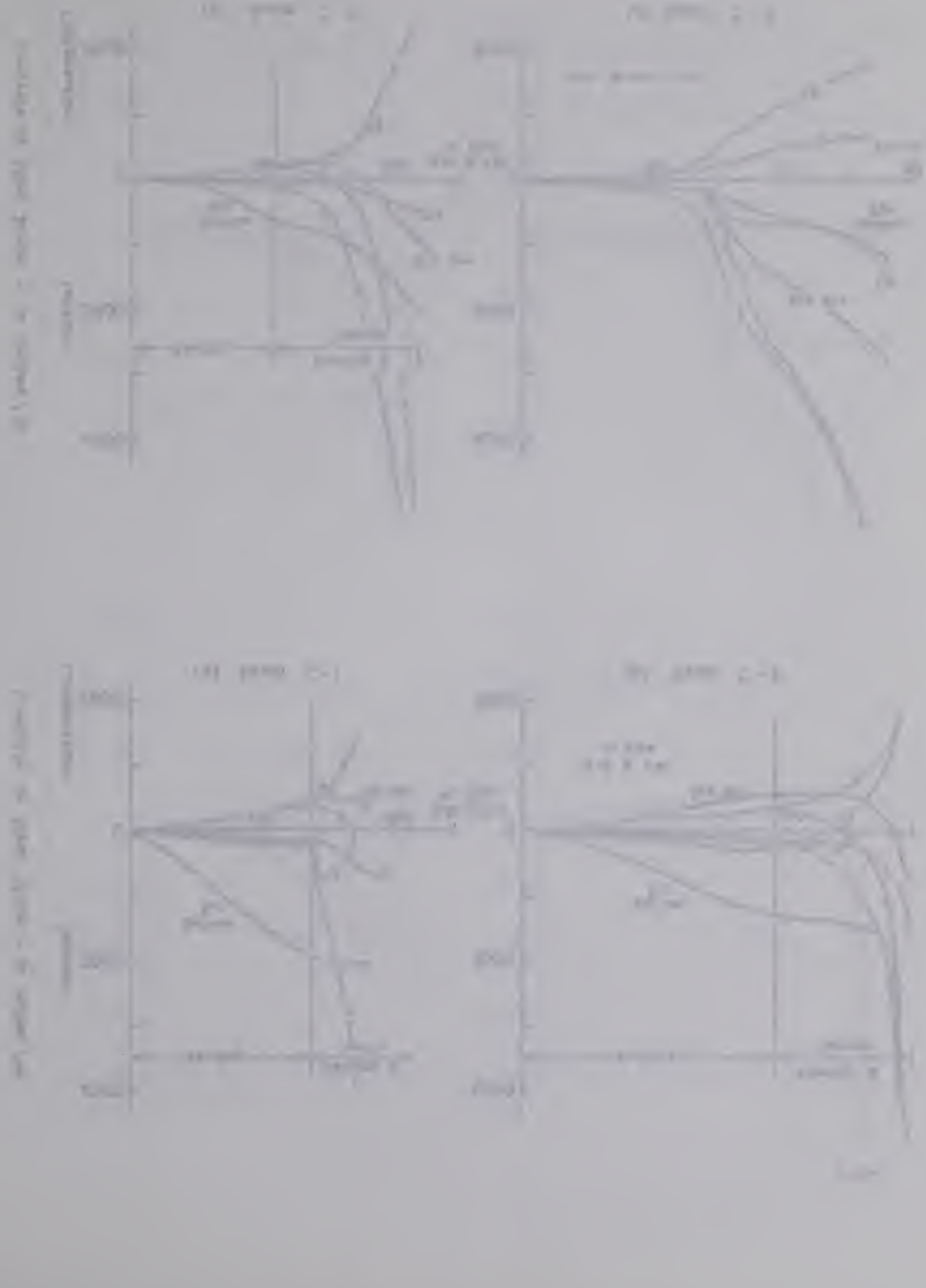


FIGURE B-3 CONCRETE & STEEL STRAINS FOR BEAMS IN GROUP C



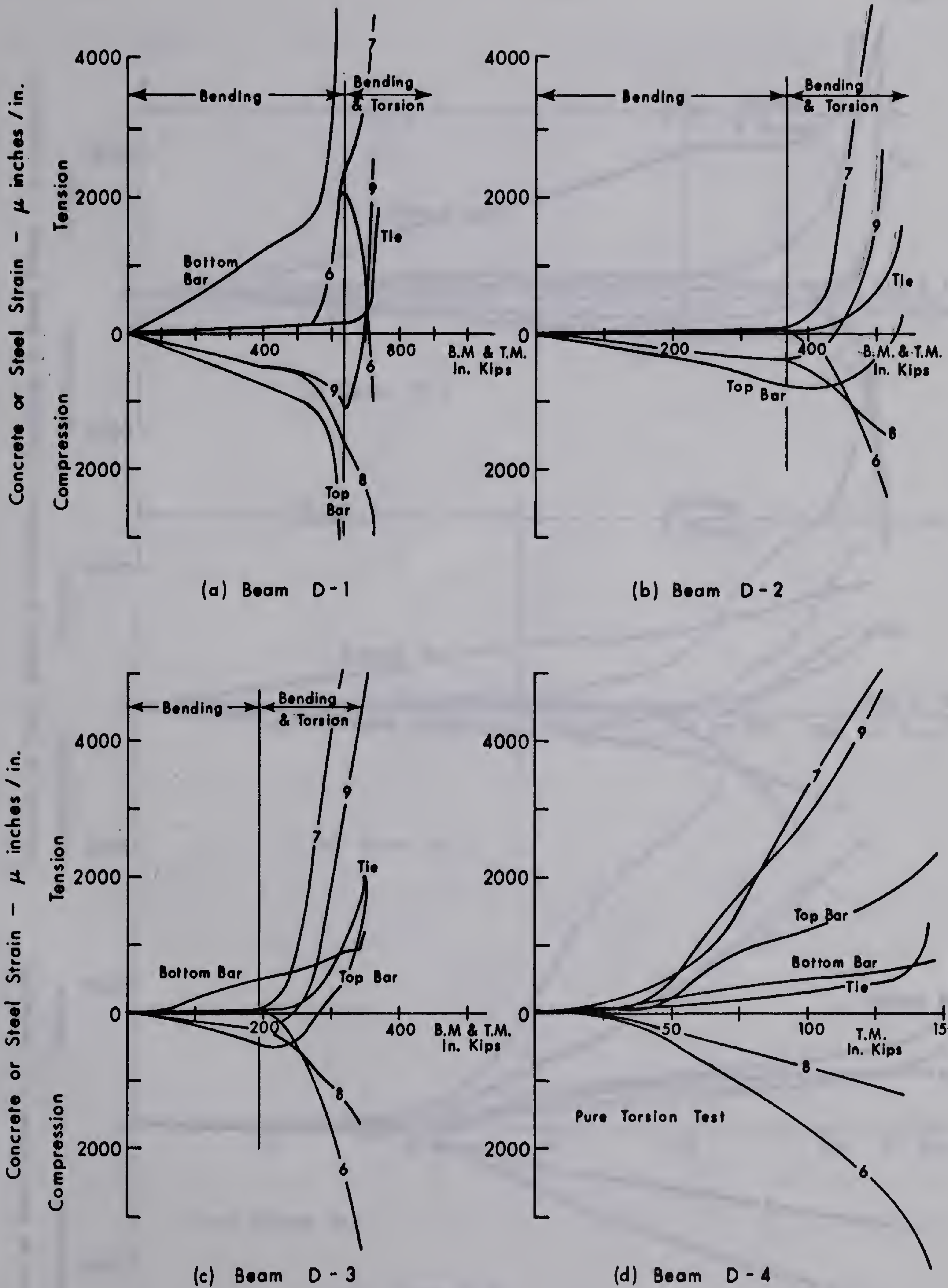


FIGURE B-4 CONCRETE & STEEL STRAINS FOR BEAMS IN GROUP D

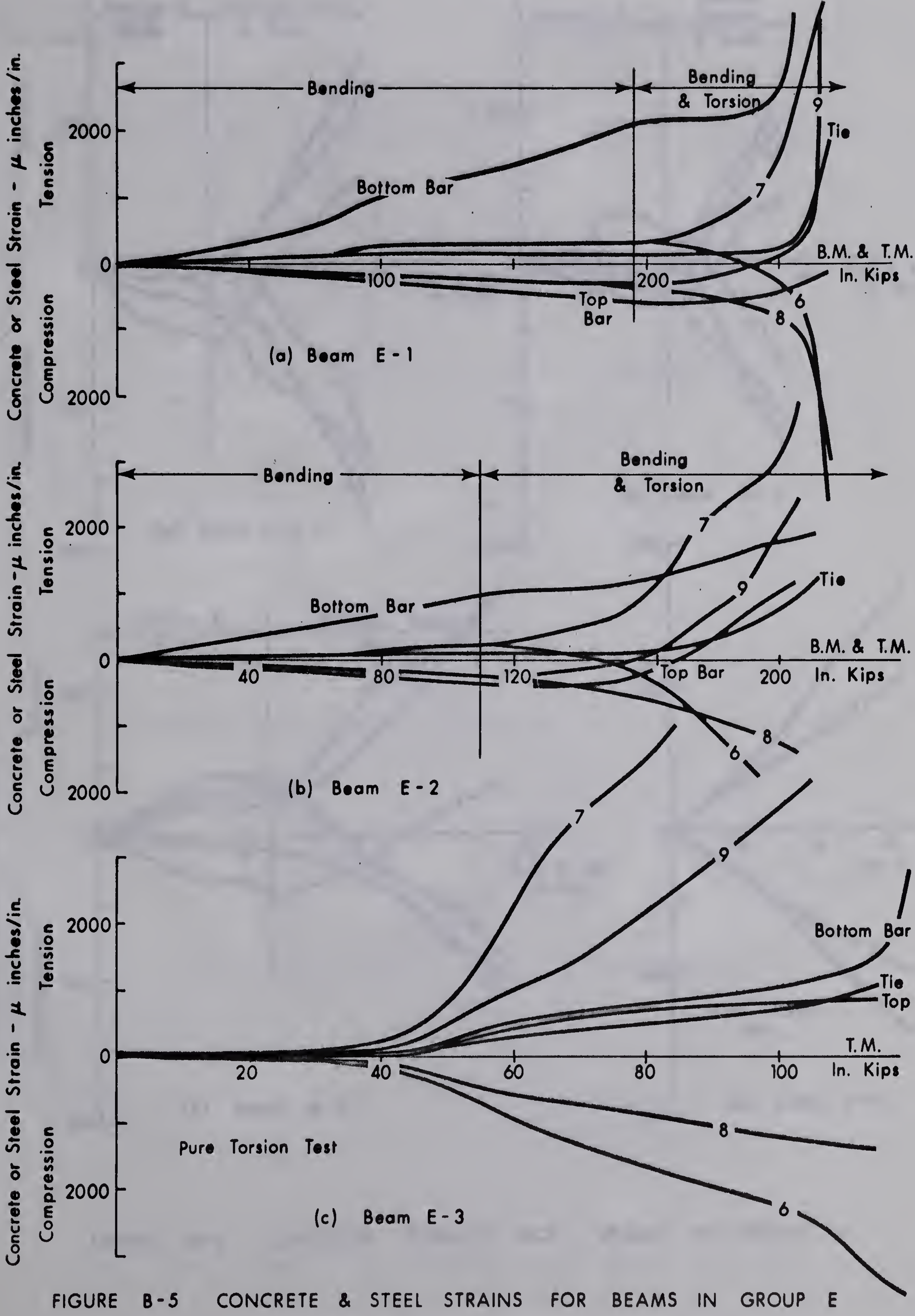


FIGURE B-5 CONCRETE & STEEL STRAINS FOR BEAMS IN GROUP E

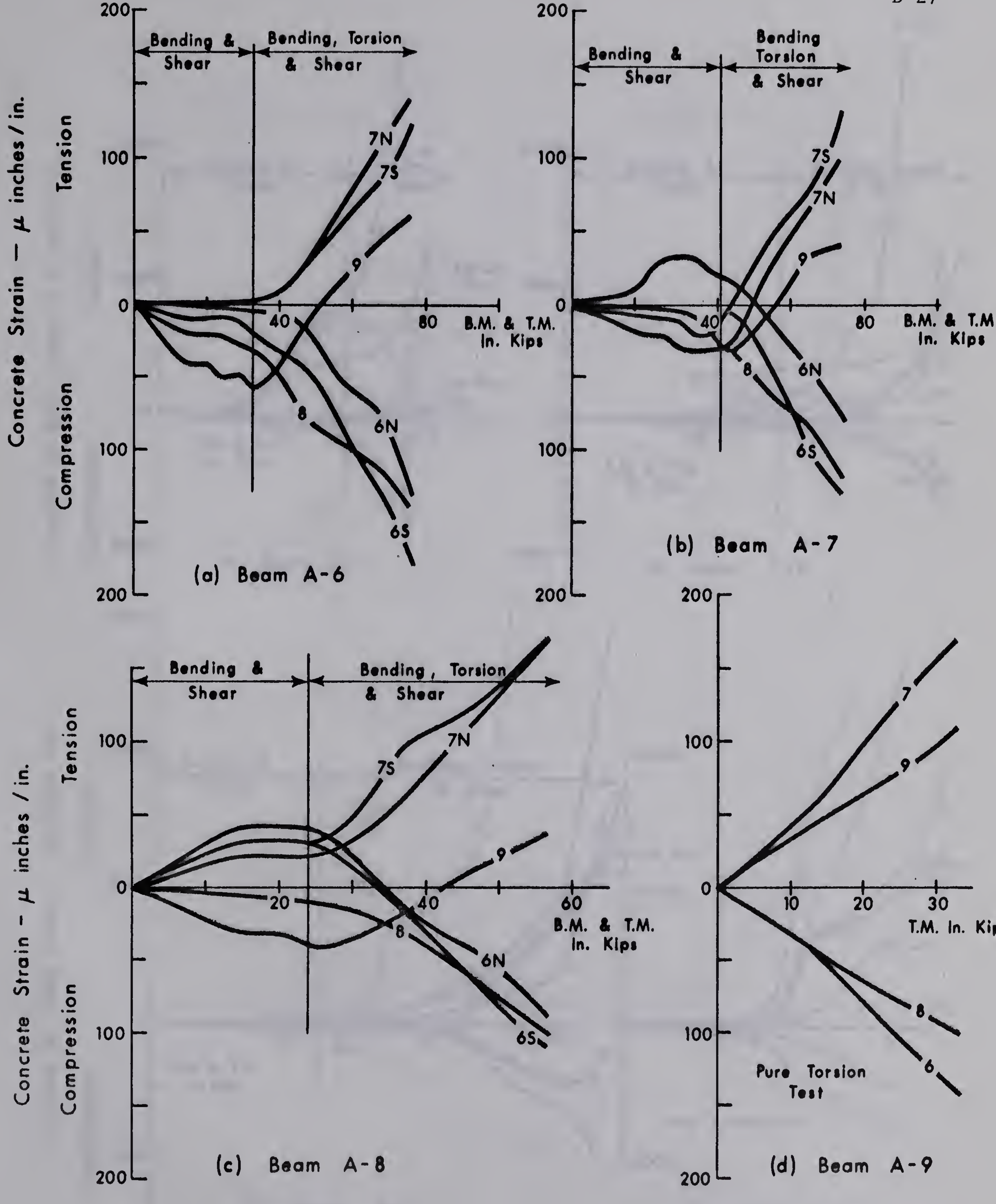
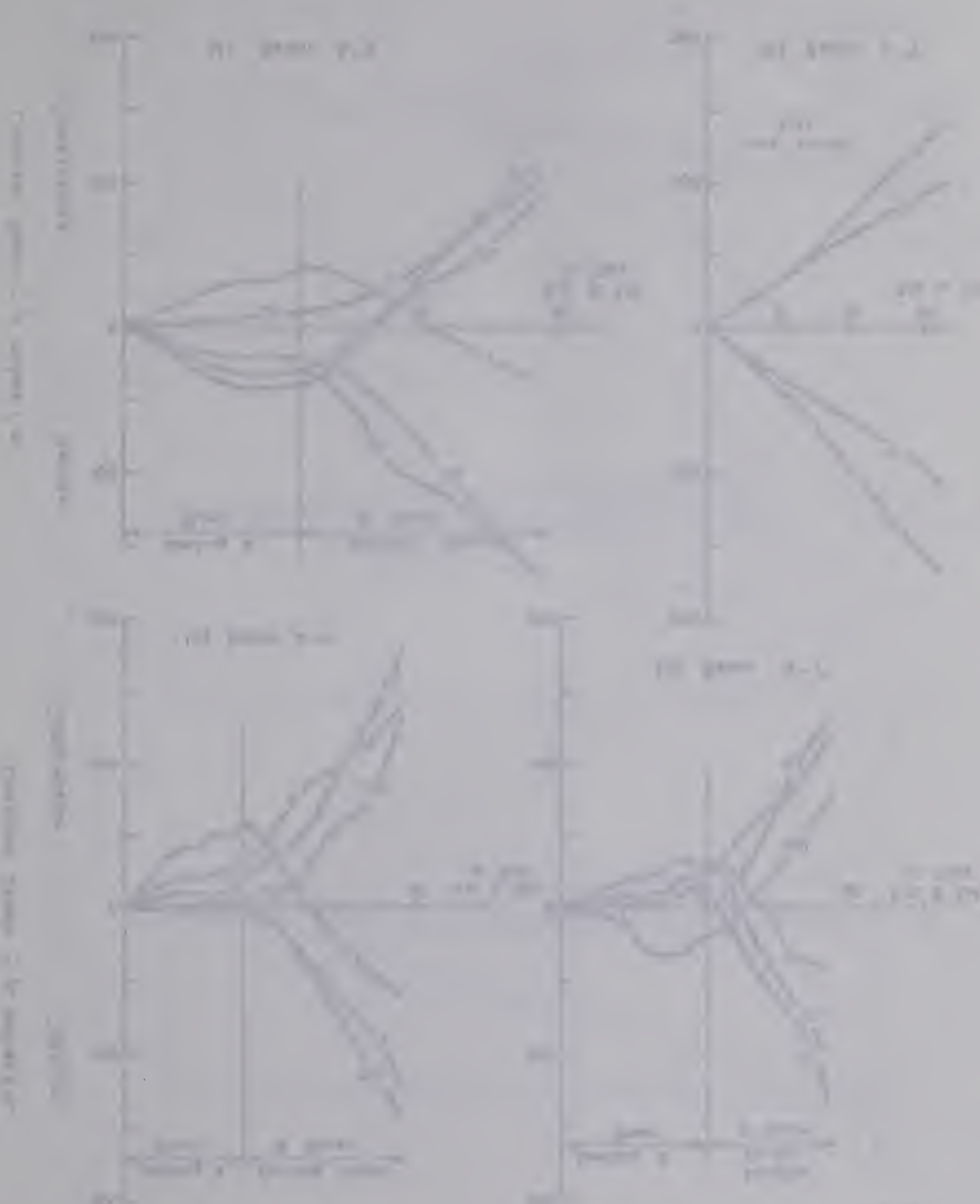


FIGURE B-6 CONCRETE STRAINS FOR BEAMS IN GROUP A



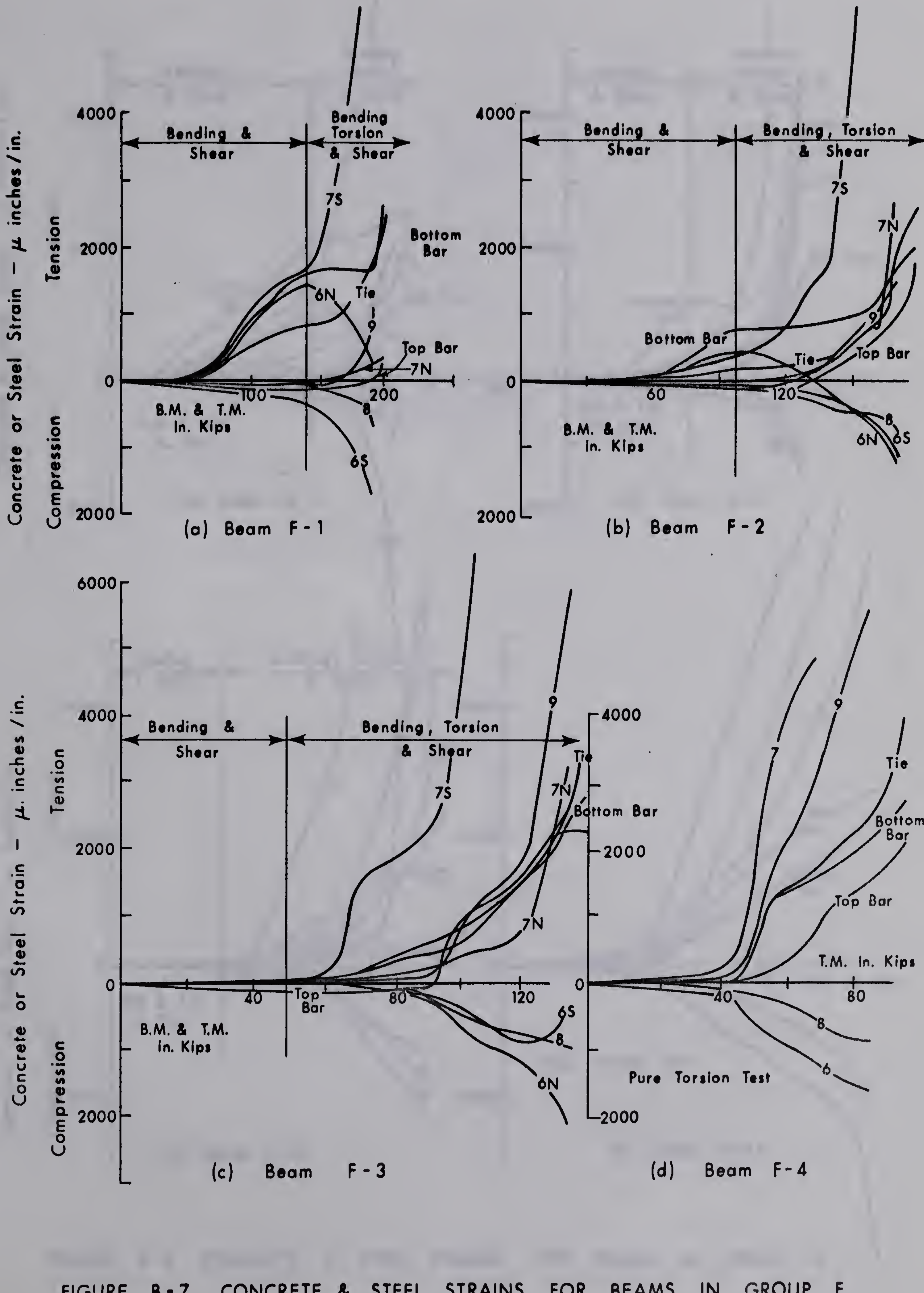


FIGURE B-7 CONCRETE & STEEL STRAINS FOR BEAMS IN GROUP F

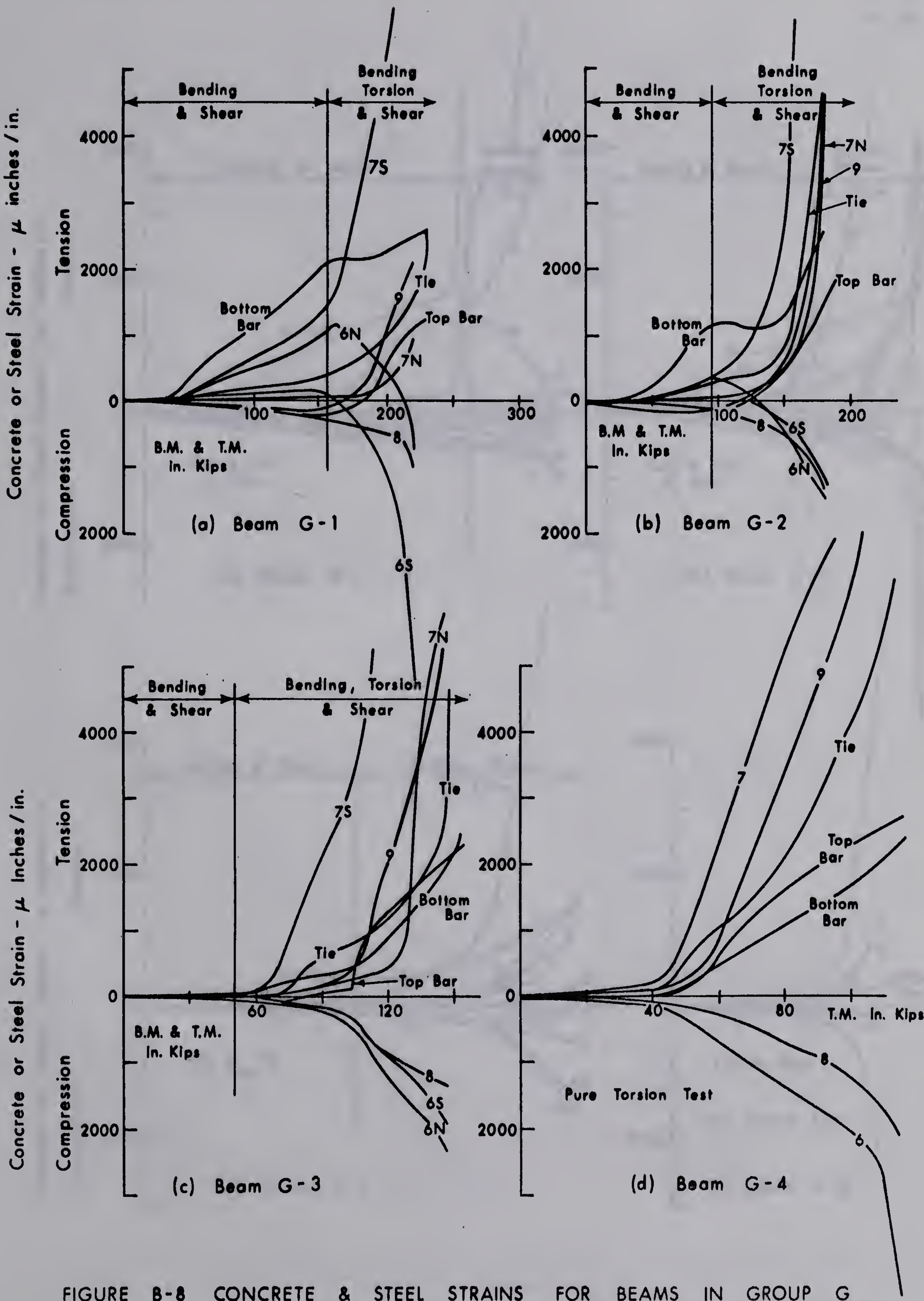


FIGURE B-8 CONCRETE & STEEL STRAINS FOR BEAMS IN GROUP G

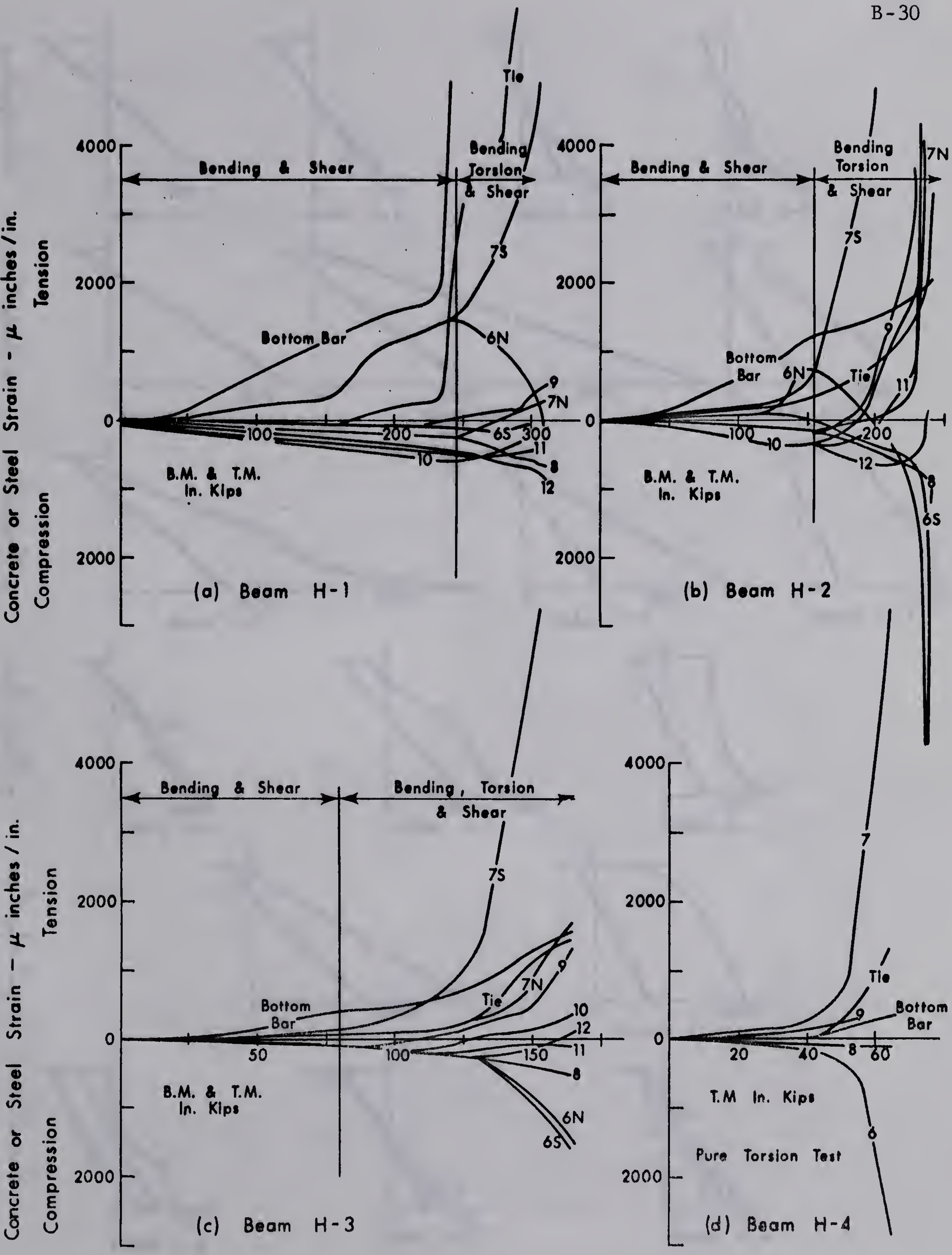


FIGURE B-9 CONCRETE & STEEL STRAINS FOR BEAMS IN GROUP H

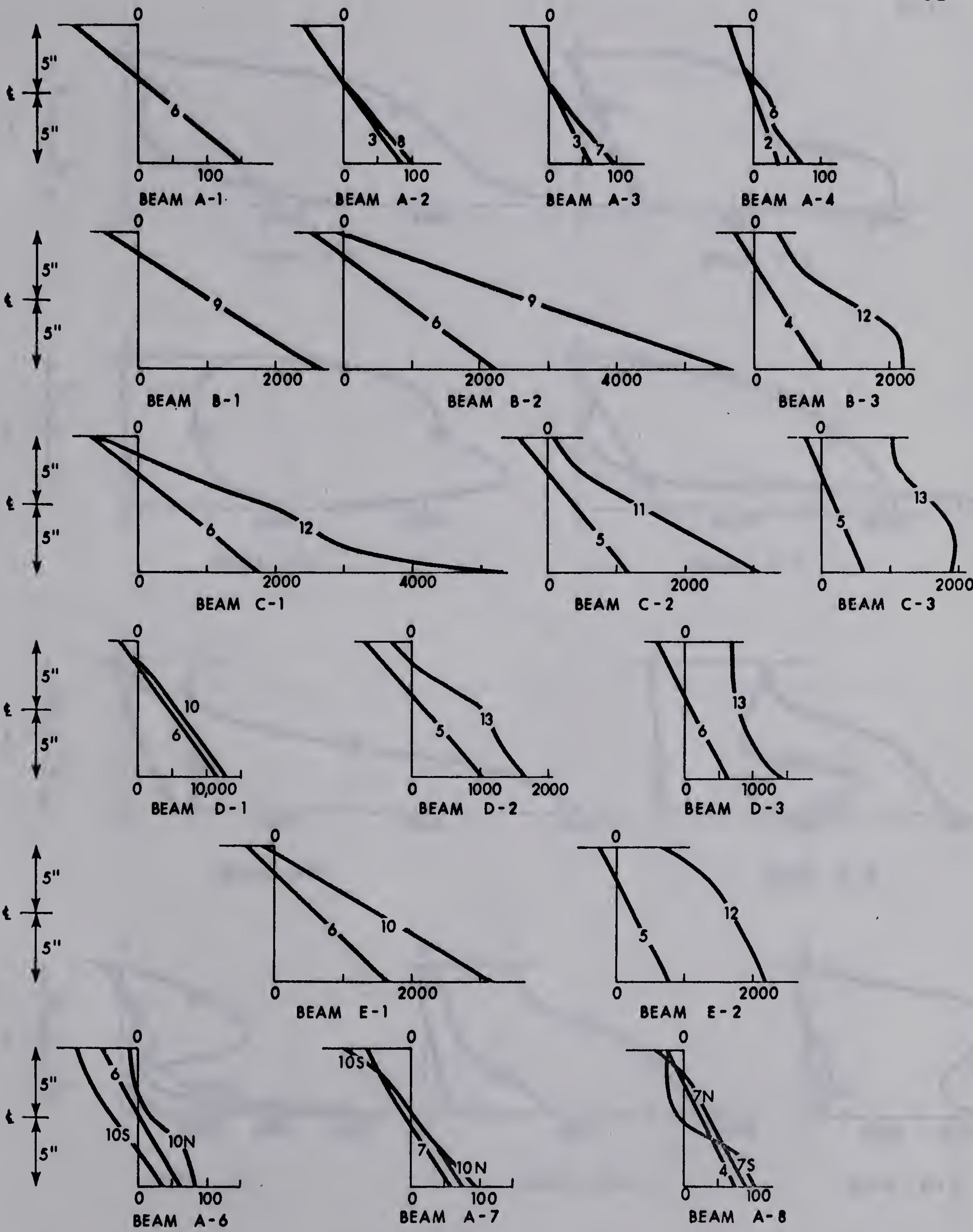


FIGURE B-10 LONGITUDINAL STRAINS IN CONCRETE - μ INCHES / IN. FOR BEAMS IN GROUPS A THROUGH E

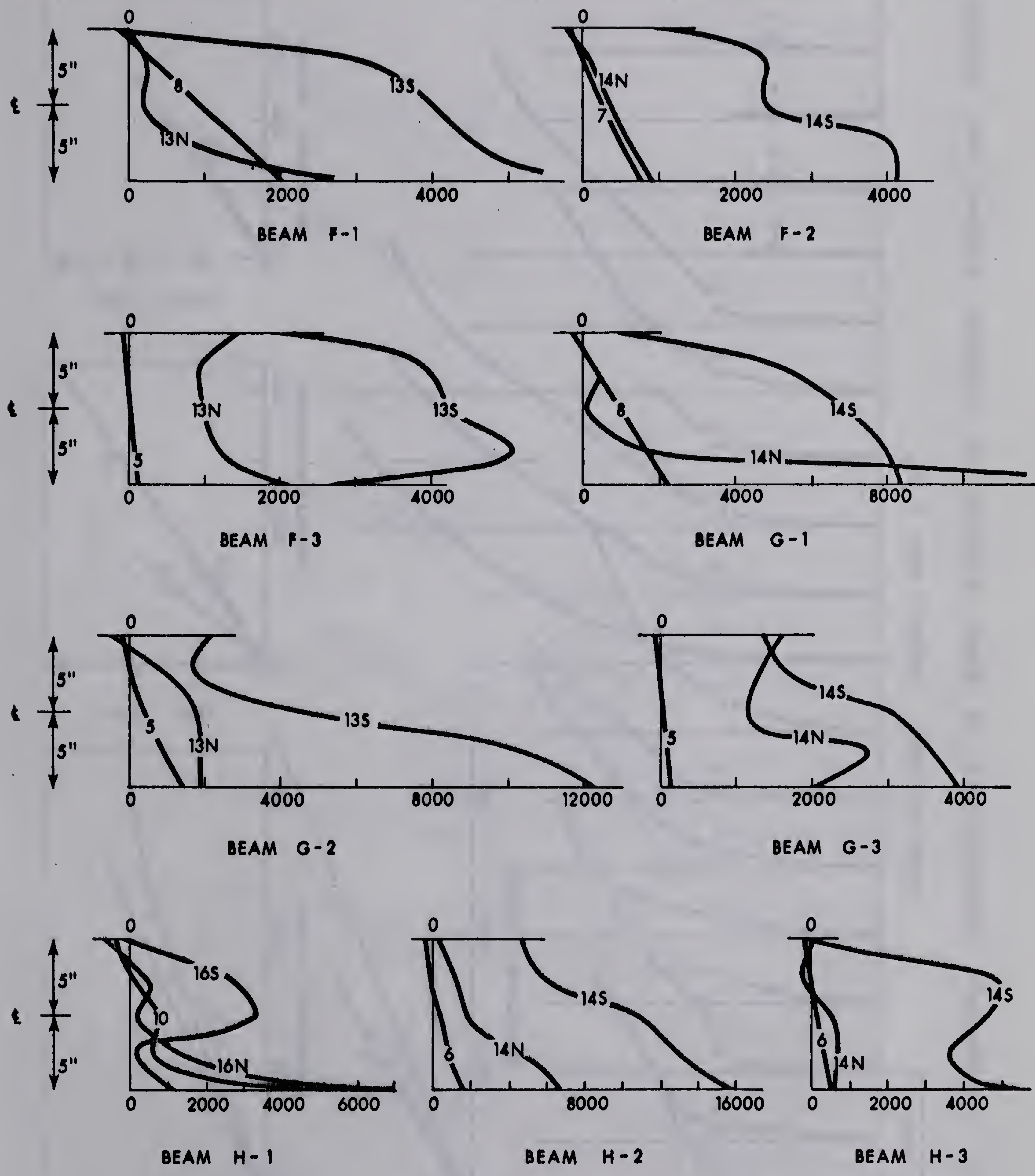


FIGURE B-11 LONGITUDINAL STRAINS IN CONCRETE - μ INCHES / IN. FOR BEAMS IN GROUPS F THROUGH H

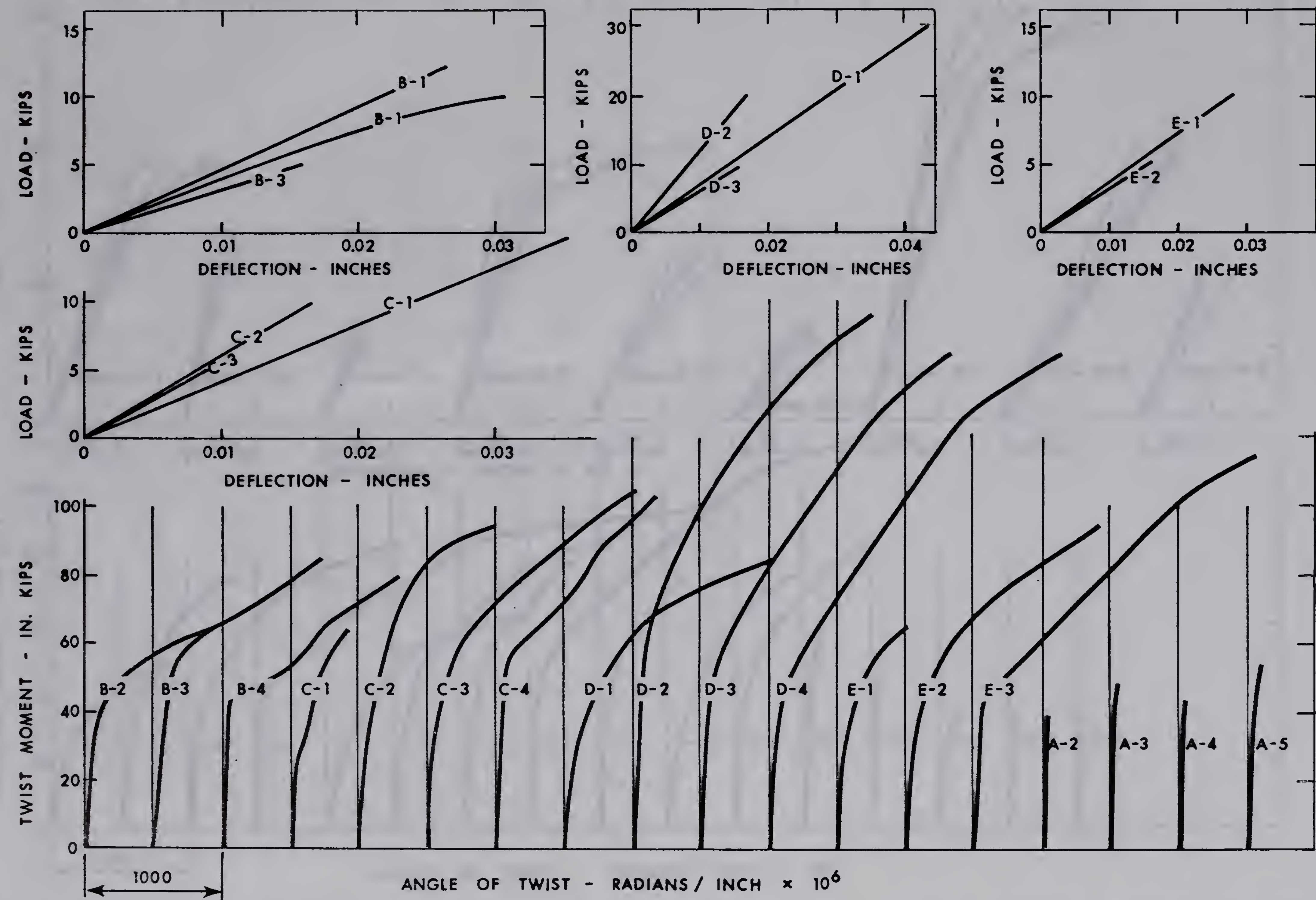


FIGURE B-12 DEFORMATION CHARACTERISTICS OF BEAMS SUBJECTED TO COMBINED BENDING AND TORSION

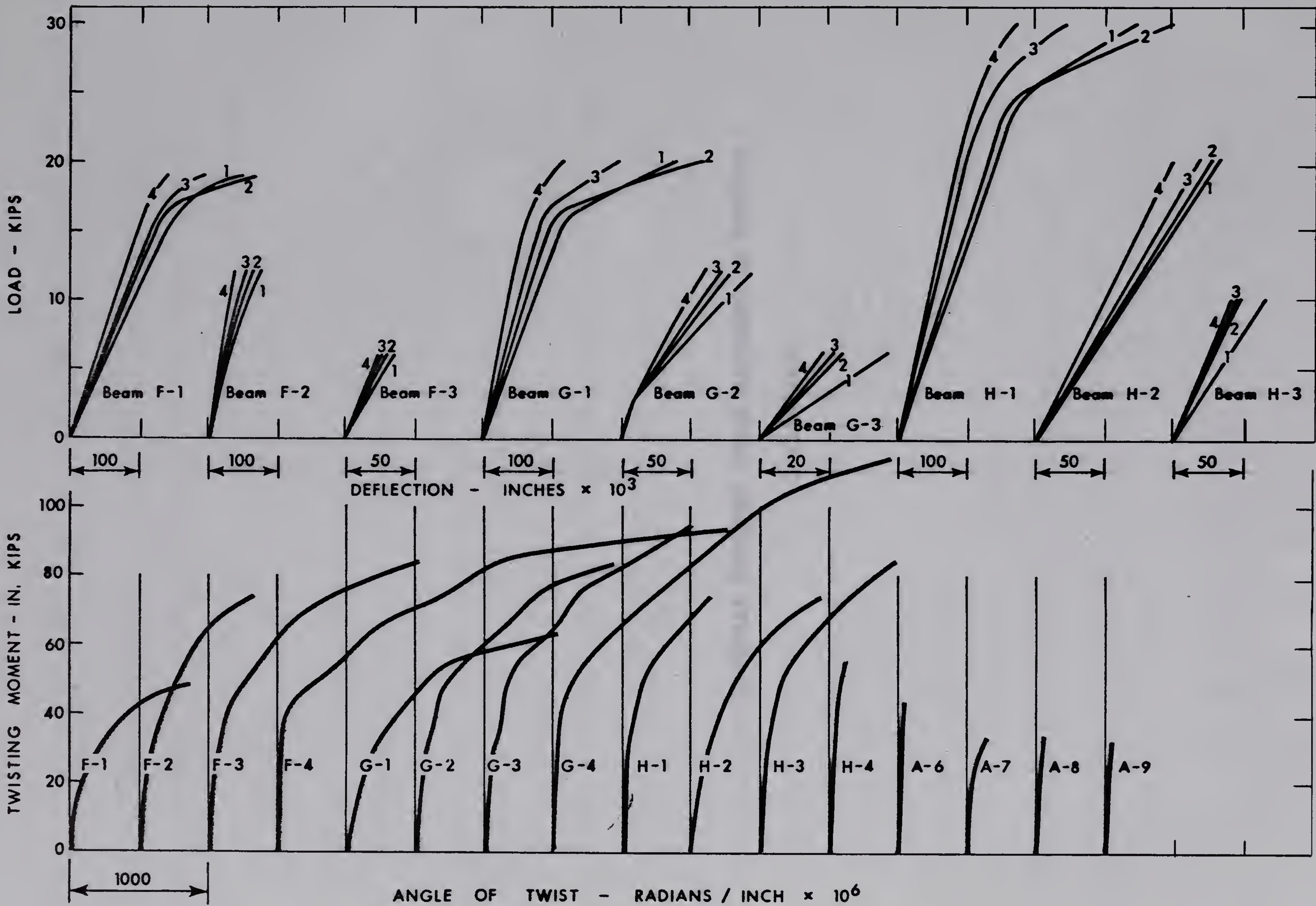


FIGURE B-13 DEFORMATION CHARACTERISTICS OF BEAMS SUBJECTED TO COMBINED BENDING, TORSION & SHEAR

APPENDIX C

FAILURE OF CONCRETE UNDER COMBINED STRESSES

APPENDIX C

FAILURE OF CONCRETE UNDER COMBINED STRESSES

C-1 Introduction

Criteria governing failure of concrete under combined stresses have received considerable interest in the past. The proposed theories can be classified in the four broad categories:

(i) maximum stress

(ii) internal friction

(iii) maximum strain

(iv) strain energy

In problems involving essentially a state of biaxial stress, the failure criterion may be expressed by a failure envelope on the Mohr's diagram. However, its use in problems involving triaxial stresses is questionable since it implies that the failure criterion is independent of the intermediate principal stress. In a state of tri-axial stress, the failure criterion may be governed by octahedral normal and shearing stresses (Bresler and Pister, 1955).

The low strength of concrete in direct tension as compared to its strength in compression or shear is an important feature of concrete. This is especially true in connection with concrete subjected to torsion which tends to induce diagonal tension in concrete.

The failure theories for concrete subjected to combined stresses have been discussed in the literature (Bresler and Pister 1955, Zia 1961, Pandit 1961). The theories commonly used are presented below.

C-2 Failure Theories

(i) Rankine's Theory

This theory assumes that the criterion of failure is attained as soon as the principal stress value reaches the strength of concrete in simple tension or compression. The failure envelope on the Mohr's diagram, FIGURE C-1, consists of a pair of straight lines parallel to shear stress axis. The circles C_1 and C_2 correspond to tension and compression failure respectively. Circle C_3 corresponds to the condition when the major and minor principal stresses attain simultaneously values equal to the compressive and tensile strength of concrete. It also represents the transition from tension to compression failure. For tension failure (circle C_1)

$$R_1 = \frac{1}{2}\sigma + \tau_o \qquad \text{(C-1)}$$

and

$$\tau = \sqrt{R_1^2 - (\frac{1}{2}\sigma)^2} \qquad \text{(C-2)}$$

where σ = normal compressive stress

τ_o = unit torsional strength of plain concrete in pure torsion

τ = apparent unit torsional strength of plain concrete under combined stresses.

For compression failure (circle C_2)

$$R_2 = f_c' - \frac{1}{2}\sigma \qquad \text{(C-3)}$$

and

$$\tau = \sqrt{R_2^2 - (\frac{1}{2}\sigma)^2} \qquad \text{(C-4)}$$

The critical case, represented by circle C_3, is obtained when R_1 and R_2 become equal. Hence Equations (C-1) and (C-3) give

$$\sigma_{cr} = f'_c - \tau_o \tag{C-5}$$

and the maximum possible apparent unit torsional strength,

$$\tau_m = \sqrt{f'_c \cdot \tau_o} \tag{C-6}$$

(ii) Coulomb's Theory

Coulomb's internal friction theory assumes that the failure is due to sliding on an oblique plane in the material. The failure envelope consists of a pair of straight lines tangent to the two circles representing states of pure shear, circle C_1 , and simple compression, circle C_2 , as shown in FIGURE C-2. The apparent torsional strength, according to this theory is given by Equation (C-7).

$$\tau^2 = \left(\frac{\tau_o^2}{\frac{1}{2} f'_c - \tau_o} + \tau_o + \frac{1}{2}\sigma\right)^2 \left(1 - \frac{2\tau_o}{f'_c}\right)^2 - \frac{1}{4}\sigma^2 \tag{C-7}$$

The optimum value of normal stress, σ , is obtained by differentiating Equation (C-7) with respect to σ and equating to zero, giving-

$$\sigma_o = \frac{f'_c\left(\frac{1}{2} f'_c - \tau_o\right)}{f'_c - \tau_o} \tag{C-8}$$

The maximum value of the apparent torsional strength is then obtained from Equation (C-7) by substituting σ_o for σ .

(iii) Mohr's Generalized Theory

Mohr's generalized internal friction theory assumes that the resistance to sliding is a function of the normal stress on the sliding plane. Hence the apparent torsional strength may be expressed as

$$\tau = c + F(\sigma) \tag{C-9}$$

The constant c and the form of the function F have to be determined

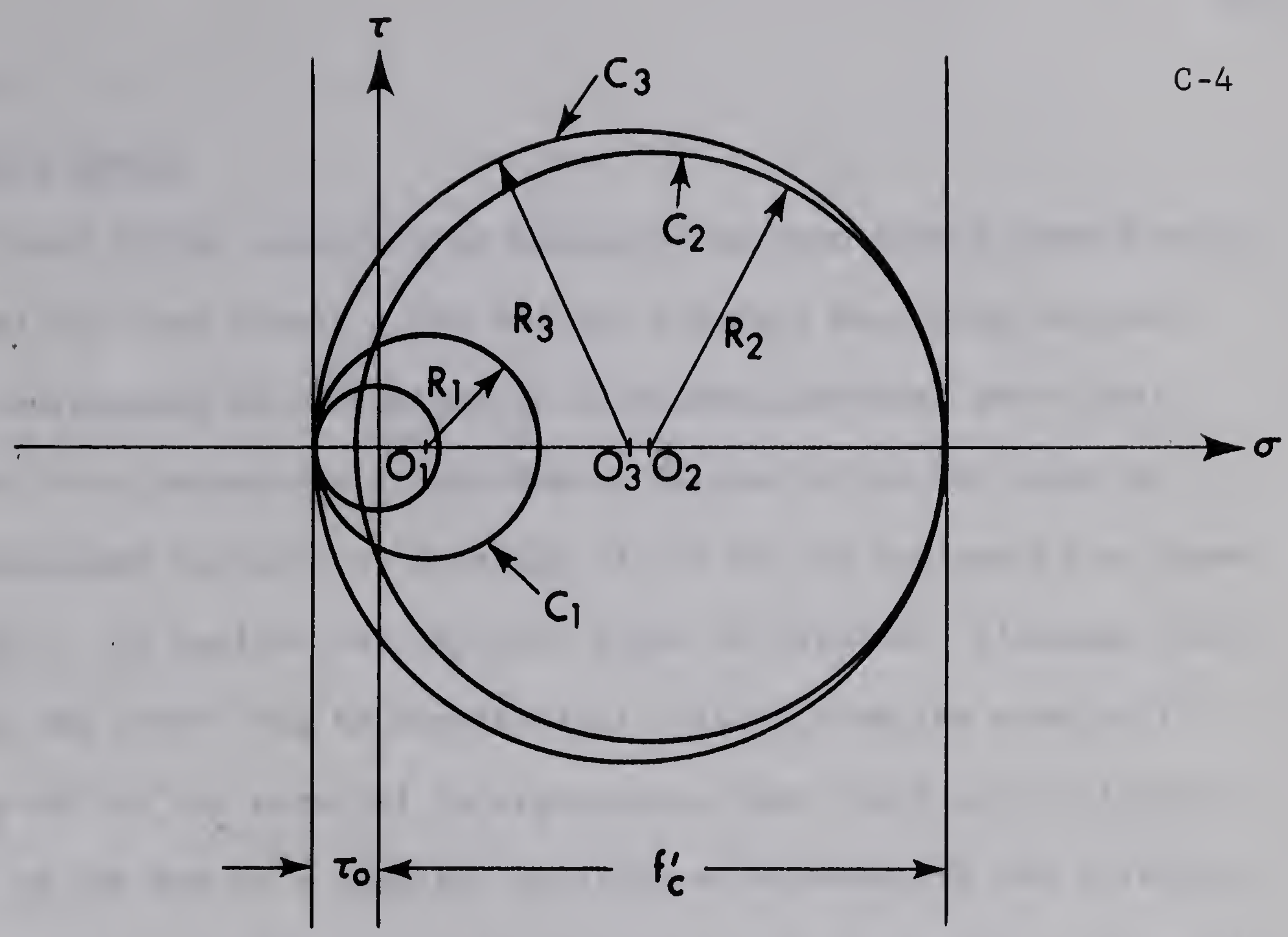


FIGURE C-1 RANKINE'S THEORY

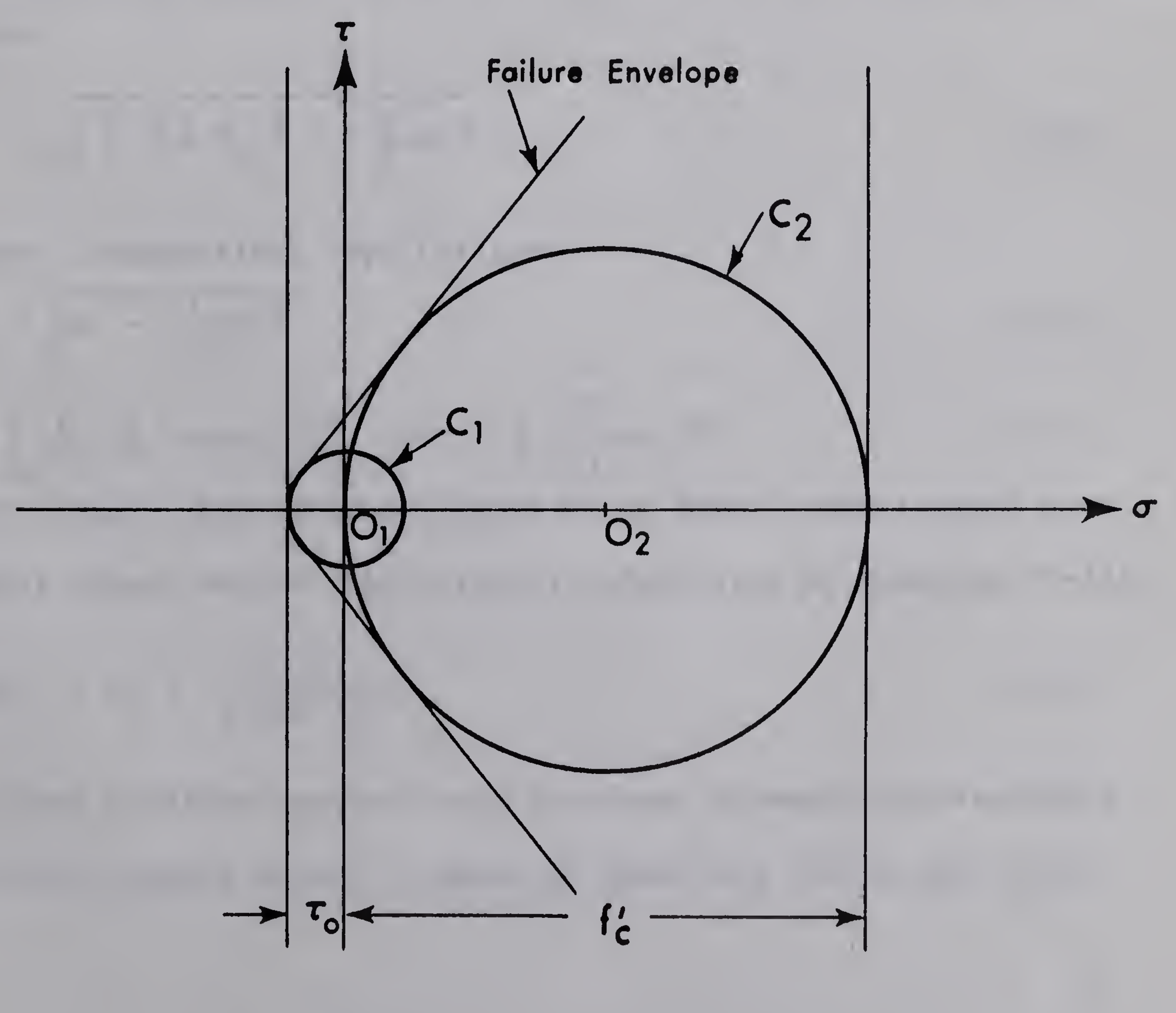


FIGURE C-2 COULOMB'S THEORY

empirically.

(iv) Cowan's Theory

Cowan (1953) combined the Rankine's maximum stress theory with the internal friction theory. The failure envelope according to this theory is represented by a combination of Rankine envelope and a pair of straight lines tangential to the Mohr's Circle C_2 for the case of simple compression inclined at an angle of 37° to the horizontal as shown in FIGURE C-3. It implies two distinct types of failure: cleavage (due to tension) and shear (due to compression). Apart from its simplicity, this theory offers the physical interpretation that the total resistance to sliding is the sum of a constant resistance representing the strength of cement paste and the frictional resistance due to aggregate, the angle of internal friction of average aggregate being 37°. For cleavage (tension) type failure

$$\tau = \sqrt{\left(\frac{1}{2}\sigma + \tau_o\right)^2 - \left(\frac{1}{2}\sigma\right)^2} \tag{C-10}$$

and for shear (compression) type failure

$$\tau = \sqrt{R^2 - \left(\frac{1}{2}\sigma\right)^2} \tag{C-11}$$

where $R = \left[\frac{1}{2} f'_c (\text{cosec } 37^\circ - 1) + \frac{1}{2}\sigma\right] \sin 37^\circ$ (C-12)

The failure changes from cleavage (tension) to shear (compression) type if the normal stress exceeds the critical value given by Equation (C-13).

$$\sigma_{cr} = f'_c - \frac{2\tau_o}{1 - \sin 37^\circ} \tag{C-13}$$

and the maximum possible apparent unit torsional strength corresponding to this critical normal stress is given by Equations (C-11) and (C-13).

(v) Cowan's Modified Theory

Zia (1961) suggested a modification to Cowan's theory to achieve a better approximation to Mohr's generalized theory (FIGURES C-3 and C-4). Equation (C-11) and (C-12) for shear (compression) failure still hold. However, the cleavage (tension) failure is governed not only by tensile strength but also by compressive strength of concrete. Thus for cleavage (tension) failure,

$$\tau = \sqrt{R^2 - \left(\frac{1}{2}\sigma\right)^2} \qquad \text{(C-14)}$$

where $$R = \tau_o + \frac{\sigma}{2}\sqrt{\left[1 - \frac{4\tau_o^2}{f_c'^2}\cot^2 26.5^o\right]} \qquad \text{(C-15)}$$

The critical normal stress at which the failure changes from one type to another is given by Equation (C-16).

$$\sigma_{cr} = \frac{f_c'(1 - \sin 37^o) - 2\tau_o}{\sqrt{\left[1 - \frac{(2\tau_o)^2}{(f_c')^2}\cot^2 26.5^o\right]} - \sin 37^o} \qquad \text{(C-16)}$$

The maximum value of apparent unit torsional strength is obtained by substituting Equation (C-16) into Equation (C-14).

(vi) Bresler and Pister's Hypothesis

Bresler and Pister (1955) suggested that the generalized failure criterion may be expressed by Equation (C-17).

$$F(I_1, I_2, I_3) = 0 \qquad \text{(C-17)}$$

where I_1, I_2 and I_3 are the three principal stress invariants given

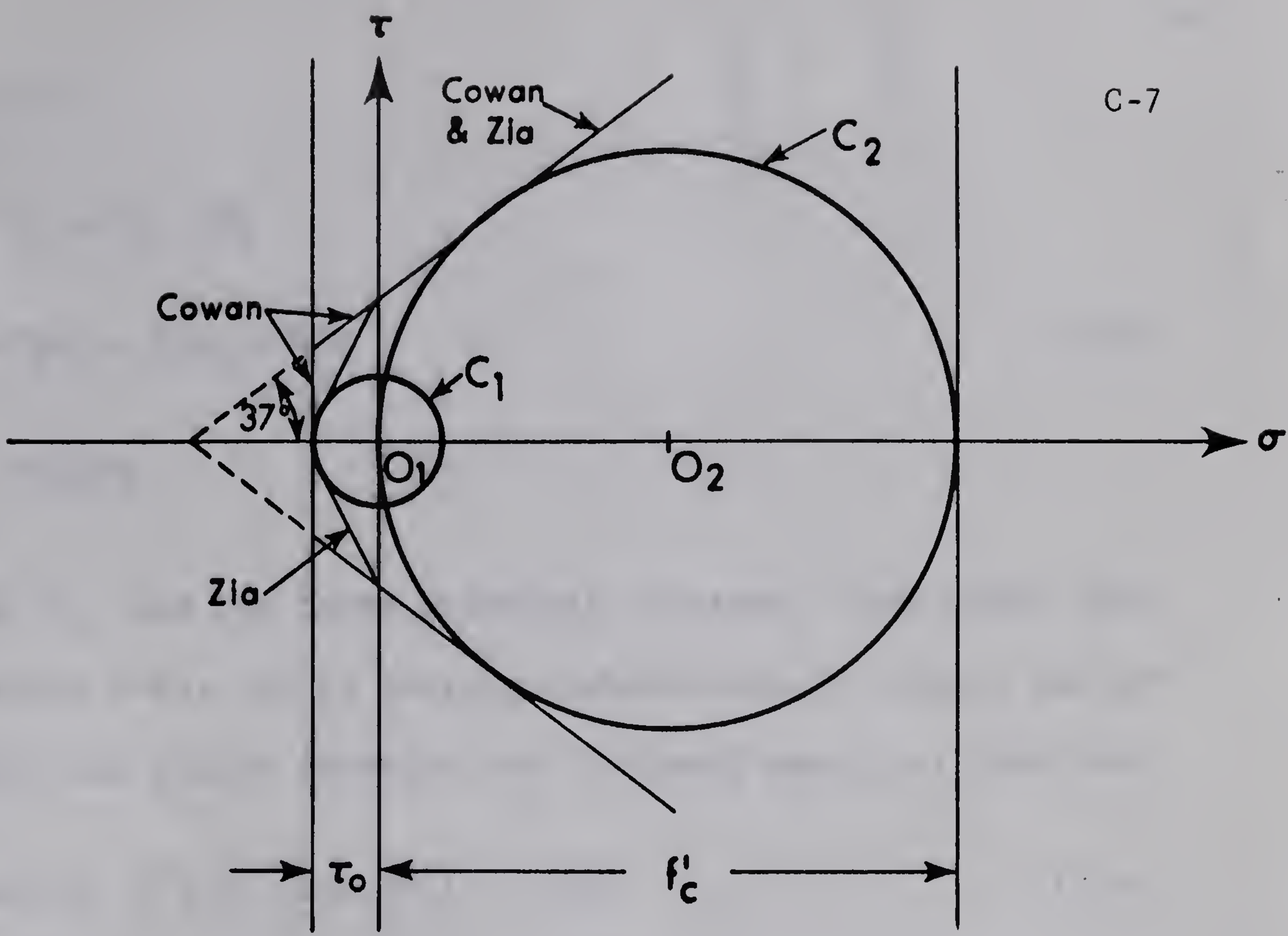


FIGURE C-3 COWAN'S THEORY & ZIA'S MODIFICATION

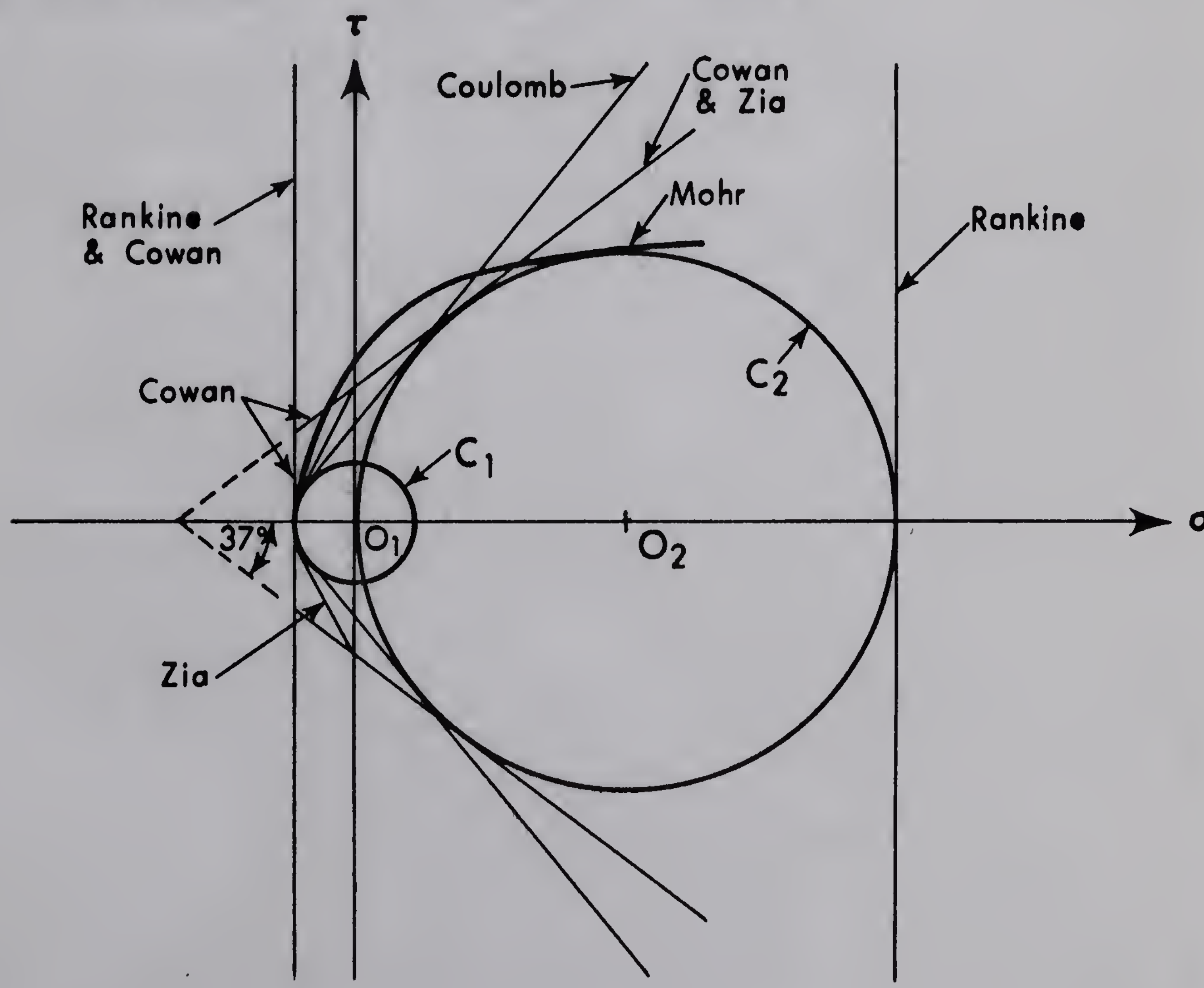


FIGURE C-4 FAILURE THEORIES

by Equations (C-18).

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1 \qquad \text{(C-18)}$$

$$I_3 = \sigma_1\sigma_2\sigma_3$$

where σ_1 σ_2 and σ_3 are the three principal stresses. From their tests on hollow cylinders subjected to varying combinations of torsion and compression, Bresler and Pister obtained the following empirical relation:

$$(-\tau_{oct}/\sigma_c) = 1.15\ (\sigma_{oct}/\sigma_c) + 0.087 \qquad \text{(C-19)}$$

where σ_{oct} and τ_{oct} are the normal and shearing octahedral stresses and σ_c is the nominal compressive strength of concrete.